Mass Spectrum in the Minimal Supersymmetric 3-3-1 model.

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Abstract

We consider the minimal supersymmetric extension of the 3-3-1 model. We study the mass spectra of this model in the fermionic and gauge bosons sectors without the antisextet. We also present some phenomenological consequences of this model at colliders such as Large Hadron Collider (LHC) and International Linear Collider(ILC).

PACS number(s): 12.60. Jv
Keywords: Supersymmetric models

1 Introduction

The models based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ (called 3-3-1 models) [1, 2, 3] provide possible solutions to some puzzles of the Standard Model (SM) such as the generation number problem, the electric charge quantization [4]. Since one generation of quarks is treated differently from the others this may lead to a natural explanation for the large mass of the top quark [5]. There is also a good candidate for SelfIinteracting Dark Matter (SIDM) since there are two Higgs bosons, one scalar and one pseudoscalar, which have the properties of candidates for dark matter like stability, neutrality and that it must not overpopulate the universe [6], etc.

There are two main versions of the 3-3-1 models depending on the embedding of the charge operator in the $SU(3)_L$ generators,

$$\frac{Q}{e} = \frac{1}{2}(\lambda_3 - \vartheta\lambda_8) + N\ I, \qquad (1)$$

where the ϑ parameter defines two different representation contents, N denotes the $U(1)_N$ charge and λ_3, λ_8 are the diagonal generators of $SU(3)$.

In the minimal version, with $\vartheta = \sqrt{3}$, the charge conjugation of the right-handed charged lepton for each generation is combined with the usual $SU(2)_L$ doublet left-handed leptons components to form an $SU(3)$ triplet $(\nu, l, l^c)_L$. No extra leptons are needed in the mentioned model, and we shall call such model as minimal 3-3-1 model. There are also another possibility where the triplet $(\nu, l, L^c)_L$ where L is an extra charged leptons wchich do not mix with the known leptons [7]. We want to remind that there is no right-handed (RH) neutrino in both model. There exists another interesting possibility, where ($\vartheta = 1/\sqrt{3}$) a left-handed anti-neutrino to each usual $SU(2)_L$ doublet is added to form an $SU(3)$ triplet $(\nu, l, \nu^c)_L$, and this model is called the 3-3-1 model with RH neutrinos. The 3-3-1 models have been studied extensively over the last decade.

The supersymmetric version of the 3-3-1 model minimal was done Refs. [8, 9] (MSUSY331) while the version with right-handed neutrinos [3] has already been constructed in Ref. [10, 11] (SUSY331RN), while the Supersymmetric economical 3-3-1 model with RH has been presented recently [12](SUSYECO331).

Recently we have already constructed all the spectrum from the scalar sector from the MSUSY331 model [13]. All the results obtained on that article are in agreement with the experimental limits. On this article we want to present the results about the masses in the fermion's sector and in the gauge boson's sector.

This paper is organized as follows. In Sec. 2 we review the minimal supersymmetric 331 model while in Sec. 4 we show how we can define one R-parity in our model such that the neutrino's get their masses and keeping the proton's safe. While in Sections (3) and (5) we present some phenomenological consequences of this model to the colliders physics. While on Sec. (6) we present the mass values of all the fermions and gauge bosons of this model. Finally, the last section is devoted to our conclusions. In Appendix A we present the Lagrangian of this model in terms of the fields.

2 Minimal Supersymmetric 3-3-1 model (MSUSY331).

On this Section, we present our model. We start to introduce the minimal set of particle necessary to get the supersymmetric version of model given at Ref. [7]. After the introduction of the particle content of our model we put them in the superfields, see Sec.(2.2), and then we construct the full

Lagrangian of our model in Sec.(2.3). Then we show the pattern of the symmetry breaking of the model at Sec.(2.4).

2.1 Particle Content

In the nonsupersymmetric 3-3-1 model [2] the fermionic representation content is as follows: left-handed leptons $L_{aL} = (\nu_a, l_a, l^c_a)_L \sim (\mathbf{1}, \mathbf{3}, 0)$, $a = e, \mu, \tau$; left-handed quarks $Q_{\alpha L} = (d_\alpha, u_\alpha, j_\alpha)_L \sim (\mathbf{3}, \mathbf{3}^*, -1/3)$, $\alpha = 1, 2$, $Q_{3L} = (u_3, d_3, J)_L \sim (\mathbf{3}, \mathbf{3}, 2/3)$; and in the right-handed components we have the usual quarks $u^c_{iL} \sim (\mathbf{3}^*, \mathbf{1}, -2/3), d^c_{iL} \sim (\mathbf{3}^3, \mathbf{1}, 1/3)$, $i = 1, 2, 3$, and the exotic quarks $j^c_{\alpha L} \sim (\mathbf{3}^*, \mathbf{1}, 4/3), J^c_L \sim (\mathbf{3}^*, \mathbf{1}, -5/3)$, they have charge $-(4/3)e$ and $(5/3)e$ respectivelly. The minimal scalar representation content is formed by three scalar triplets: $\eta \sim (\mathbf{1}, \mathbf{3}, 0) = (\eta^0, \eta^-_1, \eta^+_2)^T$; $\rho \sim (\mathbf{1}, \mathbf{3}, +1) = (\rho^+, \rho^0, \rho^{++})^T$ and $\chi \sim (\mathbf{1}, \mathbf{3}, -1) = (\chi^-, \chi^{--}, \chi^0)^T$.

Now, we introduce the minimal set of particles in order to implement the supersymmetry [14]. We have to introduce the sleptons the superpartners of the leptons and the squarks related to the quarks, both are scalars. Therefore in the supersymmetric version of this model [8, 9], the fermionic content is the same as in the nonsupersymmetric 331 model and we have to add their supersymmetric partners $\tilde{L}_{aL}$, $\tilde{Q}_{\alpha L}$, $\tilde{Q}_{3L}$, $\tilde{u}^c_{iL}$, $\tilde{d}^c_{iL}$, $\tilde{j}^c_{\alpha L}$ and $\tilde{J}^c_L$. We have also to introduce the higgsinos the supersymmetric partner of the scalars of the model and the minimal higgsinos are given by $\tilde{\eta}$, $\tilde{\rho}$ and $\tilde{\chi}$. However, we have to introduce, the followings extras scalars η', ρ', χ' and their higgsinos $\tilde{\eta}'$, $\tilde{\rho}'$ and $\tilde{\chi}'$, in order to to cancel chiral anomalies generated by $\tilde{\eta}$, $\tilde{\rho}$ and $\tilde{\chi}$.

Concerning the gauge bosons and their superpartners the gauginos. We denote the gluons by g^b, the respective superparticles, the gluinos, are denoted by λ^b_C, with $b = 1, \ldots, 8$; and in the electroweak sector we have V^b, the gauge boson of $SU(3)_L$, and their gauginos λ^b_A; finally we have the gauge boson of $U(1)_N$, denoted by V', and its supersymmetric partner λ_B.

This is the minimal number of fields in the minimal supersymmetric extension of the 3-3-1 model of Refs. [8, 9]. Summaryzing, we have in the 3-3-1 supersymmetric model the following superfields: $\hat{L}_{e,\mu,\tau}$, $\hat{Q}_{1,2,3}$, $\hat{\eta}$, $\hat{\rho}$, $\hat{\chi}$; $\hat{\eta}'$, $\hat{\rho}'$, $\hat{\chi}'$; $\hat{u}^c_{1,2,3}$, $\hat{d}^c_{1,2,3}$, $\hat{J}$ and $\hat{j}_{1,2}$, i.e., 21 chiral superfields, and 17 vector superfields: $\hat{V}^a$, $\hat{V}^\alpha$ and $\hat{V}'$. In the Minimal Supersymmetric Standard Model (MSSM) [14, 15, 16, 17, 18, 19, 20, 21, 22] there are 14 chiral superfields and 12 vector superfields.

2.2 Superfields

The superfields formalism is useful in writing the Lagrangian which is manifestly invariant under the supersymmetric transformations [23] with fermions and scalars put in chiral superfields while the gauge bosons in vector superfields. As usual the superfield of a field ϕ will be denoted by $\hat{\phi}$ [14]. The chiral superfield of a multiplet ϕ is denoted by [23]

$$\begin{aligned} \hat{\phi} \equiv \hat{\phi}(x, \theta, \bar{\theta}) &= \tilde{\phi}(x) + i\, \theta\sigma^m\bar{\theta}\, \partial_m\tilde{\phi}(x) + \frac{1}{4}\, \theta\theta\, \bar{\theta}\bar{\theta}\, \partial^m\partial_m\tilde{\phi}(x) \\ &+ \sqrt{2}\, \theta\phi(x) + \frac{i}{\sqrt{2}}\, \theta\theta\, \bar{\theta}\bar{\sigma}^m\partial_m\phi(x) + \theta\theta\, F_\phi(x), \end{aligned} \quad (2)$$

while the vector superfield is given by

$$\hat{V}(x, \theta, \bar{\theta}) = -\theta\sigma^m\bar{\theta}V_m(x) + i\theta\theta\bar{\theta}\overline{\tilde{V}}(x) - i\bar{\theta}\bar{\theta}\theta\tilde{V}(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x). \quad (3)$$

The fields F and D are auxiliary fields which are needed to close the supersymmetric algebra and eventually will be eliminated using their motion equations.

In the nonsupersymmetric 3-3-1 model to give arbitrary mass to the leptons we have to introduce one scalar antisextet $S \sim (\mathbf{1}, \mathbf{6}^*, 0)$.We can avoid the introduction of the antisextet by adding a charged lepton transforming as a singlet. Notwithstanding, here we will omit both the antisextet, we are going to show in Sec.6 all the fermions and gauge bosons of the model get their masses only with three triplets and three antitriplets in agreement with [24].

2.3 The Lagrangian

On this subsection we will write only the lagrangian in the terms of superfields of the model. The Lagrangian of the model has the following form

$$\mathcal{L}_{331S} = \mathcal{L}_{SUSY} + \mathcal{L}_{\text{soft}}, \quad (4)$$

where $\mathcal{L}_{SUSY}$ is the supersymmetric part and $\mathcal{L}_{\text{soft}}$ the soft terms breaking explicitly the supersymmetry.

The supersymmetric term can be divided as follows

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Lepton} + \mathcal{L}_{Quarks} + \mathcal{L}_{Gauge} + \mathcal{L}_{Scalar}, \quad (5)$$

The fermion's lagrangian is given by

$$
\begin{aligned}
\mathcal{L}_{\text{Lepton}} &= \int d^4\theta \; \left[\hat{\bar{L}}_a e^{2g\hat{V}} \hat{L}_a \right], \\
\mathcal{L}_{\text{Quarks}} &= \int d^4\theta \; \left\{ \hat{\bar{Q}}_\alpha e^{\left[2g_s\hat{V}_C + 2g\hat{\bar{V}} + g'\left(-\frac{1}{3}\right)\hat{V}'\right]} \hat{Q}_\alpha + \hat{\bar{Q}}_3 e^{\left[2g_s\hat{V}_C + 2g\hat{V} + g'\left(\frac{2}{3}\right)\hat{V}'\right]} \hat{Q}_3 \right. \\
&+ \hat{\bar{u}}^c_i e^{\left[2g_s\hat{\bar{V}}_C + g'\left(-\frac{2}{3}\right)\hat{V}'\right]} \hat{u}^c_i + \hat{\bar{d}}^c_i e^{\left[2g_s\hat{\bar{V}}_C + g'\left(\frac{1}{3}\right)\hat{V}'\right]} \hat{d}^c_i + \hat{\bar{J}}^c e^{\left[2g_s\hat{\bar{V}}_C + g'\left(-\frac{5}{3}\right)\hat{V}'\right]} \hat{J}^c \\
&+ \left. \hat{\bar{j}}^c_\alpha e^{\left[2g_s\hat{\bar{V}}_C + g'\left(\frac{4}{3}\right)\hat{V}'\right]} \hat{j}^c_\alpha \right\}. \qquad (6)
\end{aligned}
$$

where we have defined $\hat{V}_C = T^b\hat{V}^b_C$, $\hat{V} = T^b\hat{V}^b$; $\hat{\bar{V}}_C = \bar{T}^b\hat{V}^b{}_C$, $\hat{\bar{V}} = \bar{T}^b\hat{V}^b$; $T^b = \lambda^b/2$, $\bar{T}^b = -\lambda^{*b}/2$ are the generators of triplet and antitriplets representations, respectively, and λ^b are the Gell-Mann matrices.

In the gauge sector we have

$$
\begin{aligned}
\mathcal{L}_{gauge} &= \frac{1}{4}\int d^2\theta \; Tr[\mathcal{W}_C\mathcal{W}_C] + \frac{1}{4}\int d^2\theta \; Tr[\mathcal{W}_L\mathcal{W}_L] + \frac{1}{4}\int d^2\theta \mathcal{W}'\mathcal{W}' \\
&+ \frac{1}{4}\int d^2\bar{\theta} \; Tr[\bar{\mathcal{W}}_C\bar{\mathcal{W}}_C] + \frac{1}{4}\int d^2\bar{\theta} \; Tr[\bar{\mathcal{W}}_L\bar{\mathcal{W}}_L] + \frac{1}{4}\int d^2\bar{\theta}\bar{\mathcal{W}}'\bar{\mathcal{W}}' \; , \qquad (7)
\end{aligned}
$$

where $\mathcal{W}_C$, $\mathcal{W}_L$ e $\mathcal{W}'$ are fields that can be written as follows [23]

$$
\begin{aligned}
\mathcal{W}_{\zeta C} &= -\frac{1}{8g}\bar{D}\bar{D}e^{-2g\hat{V}_C}D_\zeta e^{2g_s\hat{V}_C}, \\
\mathcal{W}_{\zeta L} &= -\frac{1}{8g}\bar{D}\bar{D}e^{-2g\hat{V}}D_\zeta e^{2g\hat{V}}, \\
\mathcal{W}'_\zeta &= -\frac{1}{4}\bar{D}\bar{D}D_\zeta\hat{V}', \;\; \zeta = 1,2. \qquad (8)
\end{aligned}
$$

Finally, in the scalar sector we have

$$
\begin{aligned}
\mathcal{L}_{scalar} &= \int d^4\theta \; \left[\hat{\bar{\eta}} e^{2g\hat{V}} \hat{\eta} + \hat{\bar{\rho}} e^{\left(2g\hat{V} + g'\hat{V}'\right)} \hat{\rho} + \hat{\bar{\chi}} e^{\left(2g\hat{V} - g'\hat{V}'\right)} \hat{\chi} \right. \\
&+ \left. \hat{\bar{\eta}}' e^{2g\hat{\bar{V}}} \hat{\eta}' + \hat{\bar{\rho}}' e^{\left(2g\hat{\bar{V}} - g'\hat{V}'\right)} \hat{\rho}' + \hat{\bar{\chi}}' e^{\left(2g\hat{\bar{V}} + g'\hat{V}'\right)} \hat{\chi}' \right] + \int d^2\theta W + \int d^2\bar{\theta}\overline{W} \; , \qquad (9)
\end{aligned}
$$

here g and g' are the gauge coupling constants of $SU(3)$ and $U(1)$ respectivelly and W is the superpotential of the model.

The superpotential of our model is given by

$$W = \frac{W_2 + \bar{W}_2}{2} + \frac{W_3 + \bar{W}_3}{3}, \tag{10}$$

with W_2 having only two chiral superfields while W_3 has three chiral superfields. The terms allowed by our symmetry are

$$\begin{aligned} W_2 &= \mu_{0a}\hat{L}_{aL}\hat{\eta}' + \mu_\eta\hat{\eta}\hat{\eta}' + \mu_\rho\hat{\rho}\hat{\rho}' + \mu_\chi\hat{\chi}\hat{\chi}', \\ W_3 &= \lambda_{1abc}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{L}_{cL} + \lambda_{2ab}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{\eta} + \lambda_{3a}\epsilon\hat{L}_{aL}\hat{\chi}\hat{\rho} + f_1\epsilon\hat{\rho}\hat{\chi}\hat{\eta} + f_1'\epsilon\hat{\rho}'\hat{\chi}'\hat{\eta}' \\ &+ \kappa_{1\alpha i}\hat{Q}_{\alpha L}\hat{\rho}\hat{u}^c_{iL} + \kappa_{2\alpha i}\hat{Q}_{\alpha L}\hat{\eta}\hat{d}^c_{iL} + \kappa_{3\alpha\beta}\hat{Q}_{\alpha L}\hat{\chi}\hat{j}^c_{\beta L} \\ &+ \kappa_{4\alpha a i}\hat{Q}_{\alpha L}\hat{L}_{aL}\hat{d}^c_{iL} + \kappa_{5i}\hat{Q}_{3L}\hat{\eta}'\hat{u}^c_{iL} + \kappa_{6i}\hat{Q}_{3L}\hat{\rho}'\hat{d}^c_{iL} + \kappa_7\hat{Q}_{3L}\hat{\chi}'\hat{J}^c_L \\ &+ \xi_{1ijk}\hat{d}^c_{iL}\hat{d}^c_{jL}\hat{u}^c_{kL} + \xi_{2ij\beta}\hat{u}^c_{iL}\hat{u}^c_{jL}\hat{j}^c_{\beta L} + \xi_{3i\beta}\hat{d}^c_{iL}\hat{J}^c_L\hat{j}^c_{\beta L}. \end{aligned} \tag{11}$$

The coefficients $\mu_0, \mu_\eta, \mu_\rho$ and μ_χ have mass dimension, while all the coefficients in W_3 are dimensionless [21, 22]. To see the lagrangian of this model in terms of the fields see Appendix A.

The most general soft supersymmetry breaking terms, which do not induce quadratic divergence, where described by Girardello and Grisaru [25]. They found that the allowed terms can be categorized as follows:

- scalar mass term

$$\mathcal{L}_{SMT} = -m^2 A^\dagger A, \tag{12}$$

- gaugino mass term

$$\mathcal{L}_{GMT} = -\frac{1}{2}(M_\lambda \lambda^a \lambda^a + H.c) \tag{13}$$

- scalar interaction terms

$$\mathcal{L}_{int} = m_{ij}A_iA_j + f_{ijk}\epsilon^{ijk}A_iA_jA_k + H.c. \tag{14}$$

The terms on this case are similar with the terms allowed in the superpotential of the model we are considering, see Eq.(11).

They, also, must be consistent with the 3-3-1 gauge symmetry. These soft terms are given by

$$\mathcal{L}_{\text{soft}} = \mathcal{L}_{GMT} + \mathcal{L}^{\text{soft}}_{\text{scalar}} + \mathcal{L}_{SMT}, \tag{15}$$

where

$$\mathcal{L}_{GMT} = -\frac{1}{2}\left[m_{\lambda_C}\sum_{a=1}^{8}(\lambda^a_C\lambda^a_C) + m_\lambda\sum_{a=1}^{8}(\lambda^a_A\lambda^a_A) + m'\lambda_B\lambda_B + H.c.\right], \quad (16)$$

due this term the gauginos get theis masses at scale where SUSY is broken while their superpartners the gauge bosons are massless, for more detail about symmetry breaking inthis model see Sec.(2.4). The second term give masses to the higgsinos is written as

$$\begin{aligned}\mathcal{L}_{SMT} &= -m^2_\eta\eta^\dagger\eta - m^2_\rho\rho^\dagger\rho - m^2_\chi\chi^\dagger\chi - m^2_{\eta'}\eta'^\dagger\eta' - m^2_{\rho'}\rho'^\dagger\rho' - m^2_{\chi'}\chi'^\dagger\chi' \\ &- m^2_L\tilde{L}^\dagger_{aL}\tilde{L}_{aL} - m^2_{Q_\alpha}\tilde{Q}^\dagger_{\alpha L}\tilde{Q}_{\alpha L} - m^2_{Q_3}\tilde{Q}^\dagger_{3L}\tilde{Q}_{3L} - m^2_{u_i}\tilde{u}^{c\dagger}_{iL}\tilde{u}^c_{iL} - m^2_{d_i}\tilde{d}^{c\dagger}_{iL}\tilde{d}^c_{iL} - m^2_J\tilde{J}^{c\dagger}_L\tilde{J}^c_L \\ &- m^2_{j_\beta}\tilde{j}^{c\dagger}_{\beta L}\tilde{j}^c_{\beta L} + [k_1\epsilon\rho\chi\eta + k'_1\epsilon\rho'\chi'\eta' + H.c.], \end{aligned} \quad (17)$$

while the last term is given by

$$\begin{aligned}\mathcal{L}_{int} &= \Big[-M^2_a\tilde{L}_{aL}\eta^\dagger + \varepsilon_{0abc}\epsilon\tilde{L}_{aL}\tilde{L}_{bL}\tilde{L}_{cL} + \varepsilon_{1ab}\epsilon\tilde{L}_{aL}\tilde{L}_{bL}\eta + \varepsilon_{2a}\epsilon\tilde{L}_{aL}\chi\rho \\ &+ \tilde{Q}_{\alpha L}\left(\omega_{1\alpha i}\eta\tilde{d}^c_{iL} + \omega_{2\alpha i}\rho\tilde{u}^c_{iL} + \omega_{3\alpha aj}\tilde{L}_{aL}\tilde{d}^c_{jL} + \omega_{4\alpha\beta}\chi\tilde{j}^c_{\beta L}\right) \\ &+ \tilde{Q}_{3L}(\zeta_{1i}\eta'\tilde{u}^c_{iL} + \zeta_{2i}\rho'\tilde{d}^c_{iL} + \zeta_{3J}\chi'\tilde{J}^c_L) + \varsigma_{1ijk}\tilde{d}^c_{iL}\tilde{d}^c_{jL}\tilde{u}^c_{kL} + \varsigma_{2i\beta}\tilde{d}^c_{iL}\tilde{J}^c_L\tilde{j}^c_{\beta L} \\ &+ \varsigma_{3ij\beta}\tilde{u}^c_{iL}\tilde{u}^c_{jL}\tilde{j}^c_{\beta L} + H.c.\Big]. \end{aligned} \quad (18)$$

2.4 Breake structure from MSUSY331 to $SU(3)_C \otimes U(1)_Q$

The pattern of the symmetry breaking of the model is given by the following scheme(using the notation given at [2])

$$\begin{aligned}&\text{MSUSY331} \overset{\mathcal{L}_{soft}}{\longmapsto} \text{SU(3)}_C \otimes \text{SU(3)}_L \otimes \text{U(1)}_N \overset{\langle\chi\rangle\langle\chi'\rangle}{\longmapsto} \text{SU(3)}_C \otimes \text{SU(2)}_L \otimes \text{U(1)}_Y \\ &\overset{\langle\rho,\eta,\rho'\eta'\rangle}{\longmapsto} \text{SU(3)}_C \otimes \text{U(1)}_Q \end{aligned} \quad (19)$$

When one breaks the 3-3-1 symmetry to the $SU(3)_C \otimes U(1)_Q$, the scalars get the following vacuum expectation values (VEVs):

$$\begin{aligned}<\eta> &= \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}, \quad <\rho>=\begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}, \quad <\chi>=\begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}, \\ <\eta'> &= \begin{pmatrix} v' \\ 0 \\ 0 \end{pmatrix}, \quad <\rho'>=\begin{pmatrix} 0 \\ u' \\ 0 \end{pmatrix}, \quad <\chi'>=\begin{pmatrix} 0 \\ 0 \\ w' \end{pmatrix}, \end{aligned} \quad (20)$$

where $v = v_\eta/\sqrt{2}$, $u = v_\rho/\sqrt{2}$, $w = v_\chi/\sqrt{2}$, $v' = v_{\eta'}/\sqrt{2}$, $u' = v_{\rho'}/\sqrt{2}$ and $w' = v_{\chi'}/\sqrt{2}$. From this pattern of the symmetry breaking comes the following constraint [9]

$$V_\eta^2 + V_\rho^2 = (246 \text{ GeV})^2 \tag{21}$$

coming from M_W, where, we have defined $V_\eta^2 = v_\eta^2 + v_\eta'^2$ and $V_\rho^2 = v_\rho^2 + v_\rho'^2$. Therefore the VEV's of our model satisfy the conditions:

$$w, w' \gg v, v', u, u'. \tag{22}$$

3 Phenomenological Consequences in the lepton's and quark's sectors.

In the usual 3-3-1 model [2] the gauge bosons are defined as

$$\begin{aligned}
W_m^{\pm}(x) &= -\frac{1}{\sqrt{2}}(V_m^1(x) \mp iV_m^2(x)), \;\; V_m^{\pm}(x) = -\frac{1}{\sqrt{2}}(V_m^4(x) \pm iV_m^5(x)), \\
U_m^{\pm\pm}(x) &= -\frac{1}{\sqrt{2}}(V_m^6(x) \pm iV_m^7(x)), \;\; A_m(x) = \frac{1}{\sqrt{1+4t^2}}\left[(V_m^3(x) - \sqrt{3}V_m^8(x))t + V_m\right], \\
Z_m^0(x) &= -\frac{1}{\sqrt{1+4t^2}}\left[\sqrt{1+3t^2}V_m^3(x) + \frac{\sqrt{3}t^2}{\sqrt{1+3t^2}}V_m^8(x) - \frac{t}{\sqrt{1+3t^2}}V_m(x)\right], \\
Z_m'^0(x) &= \frac{1}{\sqrt{1+3t^2}}(V_m^8(x) + \sqrt{3}tV_m(x)),
\end{aligned} \tag{23}$$

where $t \equiv \tan\theta = \frac{g'}{g}$ and g' and g are the gauge coupling constants of $U(1)$ and $SU(3)$, respectively.

The bosons U^{--} and V^- are called bileptons because they couple to two leptons; thus they have two units of lepton number, it means $L = 2$. Here L is the total lepton number, give by $L = L_e + L_\mu + L_\tau$. This model does not conserve separate family lepton number, L_e, L_μ and L_τ but only the total lepton number L is conserved.

We can define the charged gauginos, in analogy with the gauge bosons in the MSSM, in the following way [9]

$$\begin{aligned}
\lambda_W^{\pm}(x) &= -\frac{1}{\sqrt{2}}(\lambda_A^1(x) \mp i\lambda_A^2(x)), \;\; \lambda_V^{\pm}(x) = -\frac{1}{\sqrt{2}}(\lambda_A^4(x) \pm i\lambda_A^5(x)), \\
\lambda_U^{\pm\pm}(x) &= -\frac{1}{\sqrt{2}}(\lambda_A^6(x) \pm i\lambda_A^7(x)).
\end{aligned} \tag{24}$$

The charged current interactions for the fermions, came from $\mathcal{L}_{llV}$ (first equation at Eq.(120)) and from $\mathcal{L}_{qqV}$ (first equation at Eq.(121)) we can rewrite them in the following way

$$
\begin{aligned}
\mathcal{L}_l^{CC} &= -\frac{g}{\sqrt{2}}\sum_l \left(\bar{\nu}_{lL}\gamma^m l_L W_m^+ + \bar{l}_L^c\gamma^m \nu_{lL}V_m^+ + \bar{l}_L^c\gamma^m l_L U_m^{++} + h.c.\right), \\
\mathcal{L}_q^{CC} &= -\frac{g}{2\sqrt{2}}\left[\overline{U}\gamma^m(1-\gamma_5)V_{\mathrm{CKM}}DW_m^+ + \overline{U}\gamma^m(1-\gamma_5)\zeta\mathcal{J}\mathcal{V}_m + \overline{D}\gamma^m(1-\gamma_5)\xi\mathcal{J}\mathcal{U}_m\right] \\
&+ \text{H. c.},
\end{aligned}
\tag{25}
$$

where we have defined the mass eigenstates in the following way

$$
\begin{aligned}
U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D &= \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad \mathcal{V}_m = \begin{pmatrix} V_m^+ \\ U_m^{--} \\ U_m^{--} \end{pmatrix}, \\
\mathcal{U}_m &= \begin{pmatrix} U_m^{--} \\ V_m^+ \\ V_m^+ \end{pmatrix},
\end{aligned}
\tag{26}
$$

and $\mathcal{J} = \mathrm{diag}\begin{pmatrix} J_1 & J_2 & J_3 \end{pmatrix}$. The V_{CKM} is the usual Cabibbo-Kobayashi-Maskawa mixing matrix and ξ and ζ are mixing matrices containing new unknown mixing parameters due to the presence of the exotic quarks.

We can calculate the Higgs couplings to the usual leptons on this model is given by $\mathcal{L}_{llH}$, see Eq.(128) at Appendix A, we get the following lagrangian [2]

$$
\mathcal{L}_{llH} = \frac{\lambda_{2ab}}{3}(-\bar{l}_{aR}l_{bL}\eta^0 + \bar{l}_{aR}\nu_{bL}\eta_1^- + \bar{\nu}_{aR}^c l_{aL}\eta_2^+ + H.c.), \tag{27}
$$

the coupling $\mathcal{L}_{qqH}$ is the same as get in [2] on their Eq.(13). On this model the same flavor leptons they don't couple with the neutral Higgs, therefore our lighest Higgs doesn't couples with two electrons [1] and of course it can decay in the following way $H_1^0 \to e^\pm\mu^\mp$, and their coupling is $\lambda_{2e\mu} = 10^{-3}$, see [24], due this fact our light Higgs with $m_{H_1^0} = 110,5$GeV was not detected [13] by the experiment Large Electron Positron (LEP).

[1] I would like to thanks E. Gregores that call my attention to this dangeours Higgs decay channel.

We have already showed that in the Møller scattering and in muon-muon scattering we can show that left-right asymmetries $A_{RL}(ll)$ are very sensitive to a doubly charged vector bilepton resonance but they are insensitive to scalar ones [26, 27, 28].

Similarly, we have the neutral currents coupled to both Z^0 and Z'^0 massive vector bosons, according to the Lagrangian

$$\mathcal{L}_{\nu}^{NC} = -\frac{g}{2}\frac{M_Z}{M_W}\bar{\nu}_{lL}\gamma^m\nu_{lL}\left[Z_m - \frac{1}{\sqrt{3}}\frac{1}{\sqrt{h(t)}}Z'_m\right], \tag{28}$$

with $h(t) = 1 + 4t^2$, for neutrinos and

$$\mathcal{L}_{l}^{NC} = -\frac{g}{4}\frac{M_Z}{M_W}\left[\bar{l}\gamma^m(v_l + a_l\gamma^5)lZ_m + \bar{l}\gamma^m(v'_l + a'_l\gamma^5)lZ'_m\right], \tag{29}$$

for the charged leptons, where we have defined

$$\begin{array}{rlrl} v_l = & -1/h(t), & a_l = & 1, \\ v'_l = & -\sqrt{3/h(t)}, & a'_l = & v'_l/3. \end{array}$$

We can use muon collider to discover the new neutral Z'^0 boson using the reaction $\mu e \to \mu e$ it was shown at [27, 29] that $A_{RL}(\mu e)$ asymmetry is considerably enhanced.

The Lagrangian interaction among quarks and the Z^0 is

$$\mathcal{L}_{ZQ} = -\frac{g}{4}\frac{M_Z}{M_W}\sum_i\left[\bar{\Psi}_i\gamma^m(v^i + a^i\gamma^5)\Psi_i\right]Z_m, \tag{30}$$

where $i = u, c, t, d, s, b, J_1, J_2, J_3$; with

$$\begin{array}{rlrr} v^U = & (3+4t^2)/3h(t), & a^U = & -1, \\ v^D = & -(3+8t^2)/3h(t), & a^D = & 1, \\ v^{J1} = & -20t^2/3h(t), & a^{J_1} = & 0, \\ v^{J_2} = v^{J_3} = & 16t^2/3h(t), & a^{J_2} = a^{J_3} = & 0, \end{array}$$

U, and D mean the charge $+2/3$ and $-1/3$ respectively, the same for $J_{1,2,3}$. There is also the usual QCD Lagrangian given by

$$\mathcal{L}_q^{QCD} \;=\; = g_s G^{\mu}\bar{q}\gamma_{\mu}q\ , \tag{31}$$

the lagrangians presented at Eqs.(25,28,29,30,31) are the same as appear in the 331 model [2]. In those lagrangians appear a lot of interestings phenomenologiacal studies presented at [30].

We can, also, study the following process resulting in at least three leptons coming from pp collision, throught the following reactions [2]

$$\begin{aligned} g+d &\rightarrow U^{--}+J,\ \ g+u\rightarrow U^{++}+j_{\alpha}, \\ d+\bar{d} &\rightarrow U^{++}U^{--},\ \ d+\bar{u}\rightarrow U^{--}+V^{+}. \end{aligned} \tag{32}$$

For example the first process has the following Feynmann diagrams drawing in Figs. (1,2). Similar diagrams can be drawn to the process $g+u\rightarrow U^{++}+j_{\alpha}$ (change $d\rightarrow u$ and $J\rightarrow j$).

As first results, in Fig.3 we present the differencial cross section, get from the program COMPHEP [31], to the process $gd\rightarrow JU^{--}$ as function of $cos(p1,p3)$. We have also calculated the cross section to these process as function of M_U and M_J and our results are shown in Figs.4 (left) and Figs.4 (right) respectively. In Figs. 5 left(right) we present the results on forward-baskward asymmetry as function of $M_U(M_J)$ respectivelly. Similar results can also be get to the process $g+u\rightarrow U^{++}+j_{\alpha}$.

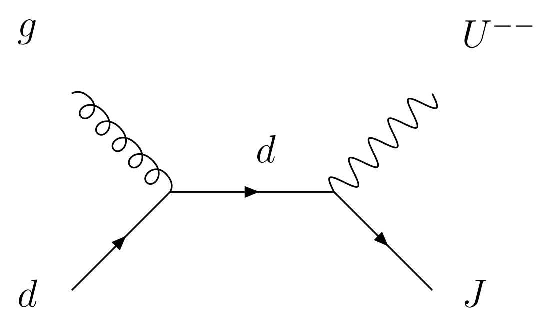

Figure 1: $gd\rightarrow U^{--}J$ exchanging quark-d.

From Eq.(31), the new gauge bosons can decay in the followings channels $U^{--}\rightarrow(\bar{J}d,\bar{u}j_{\beta},l^{-}l^{-})$ and $V^{-}\rightarrow(\bar{J}u,\bar{d}j_{\beta},l^{-}\nu)$. These decays modes are shown in Fig. 6.

The heavy quarks J, j_1 and j_2 can decay to the light quark via V^{*}/U^{*} emission to produce bilepton final states with a specific decay signature, see

[2]I would like to thanks to Alexander Belyaev that call my attention to the first process.

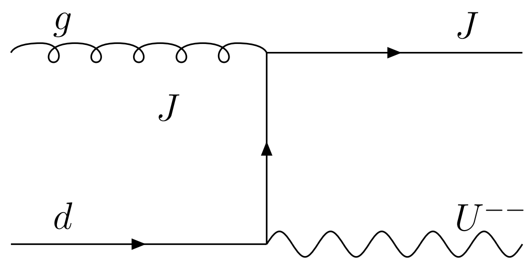

Figure 2: $gd \to U^{--}J$ exchanging quark-J.

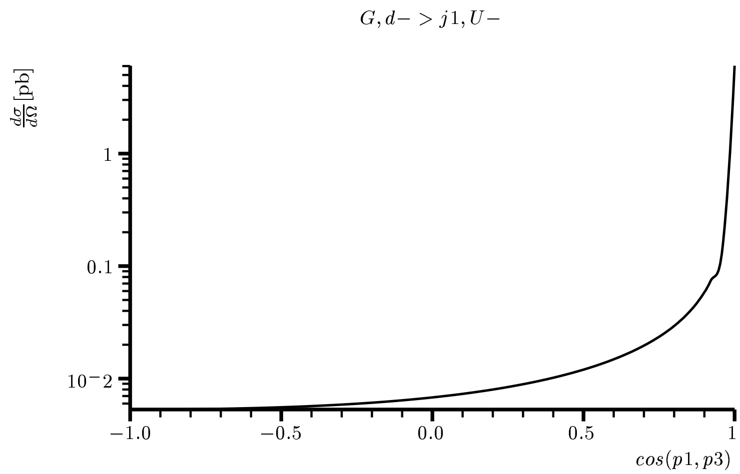

Figure 3: Differential Cross Section $gd \to JU^{--}$.

Fig. 7. Analasing these decays mode, we conclude that the J quark will decay in l^+l^+d or $l^+\nu u$ without any restrictions coming from the bosons gauge masses, because these particles are virtual on this decay. While the j quark can decay in $l^-\nu d$ and $l^-l^-\bar{u}$.

By another hand, the U decay will depend of M_U, M_J and M_{j_β}. in Tab. 1 we shown all possibles possibilities.

The width of the U boson is drawing in the Fig. 8 (left) as function of its mass. In Fig. 8 (center) we draw Γ_U versus M_J, while in Fig. 8 (right) we plot Γ_U versus M_j.

Again we divided the signals for the process $gd \to llX$ in four regions.

We also present the width of the V boson versus its mass in Fig. 9 (left)

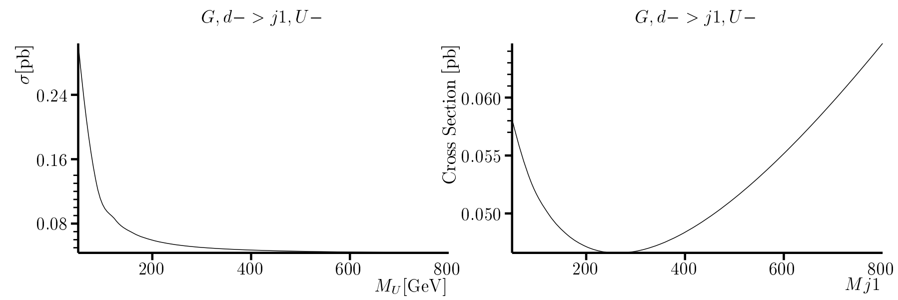

Figure 4: Total Cross Section $gd \to JU^{--}$ as function of M_U (left) and M_J (right).

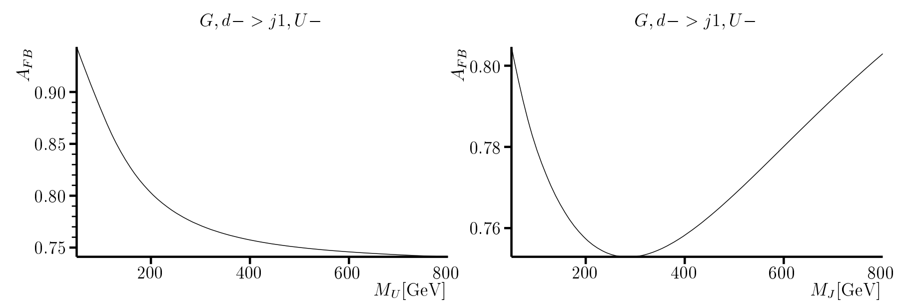

Figure 5: Asymmetry $gd \to JU^{--}$ as function of M_U (left) and M_J (right).

as function of its mass. In Fig. 9 (center) Γ_V versus M_J is shown, while in Fig. 8 (right) we plot Γ_V versus M_j.

Of course this process must be better studied as the others three processes listed at Eq.(32). These particles can be detected at Large Hadron Collider (LHC) if they really exist in nature.

There are background come mainly from the SM and from MSSM [21, 22]. The background from the SM comes from the $W^\star Z^\star$, $W^\star \gamma^\star$,$Z^\star Z^\star$ and $\bar{q}q$. Where in the SM we have the following decays for the gauge bosons $W^- \to l^- \nu_l$, $Z^0 \to l^+ l^-$ and $\gamma \to l^+ l^-$.

The $W^\star Z^\star$ and $W^\star \gamma^\star$ background are known to be the major source of background come from the SM for the three lepton channjel. The second largest background font to three leptons channel is from $q\bar{q}$ events. Finally, the remaining three leptons background which should worry about is the $Z^\star Z^\star$ jet production.

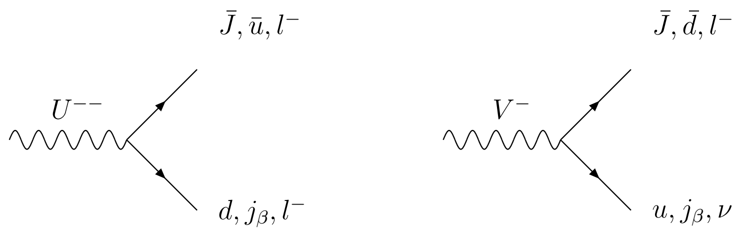

Figure 6: U and V decay in two particles

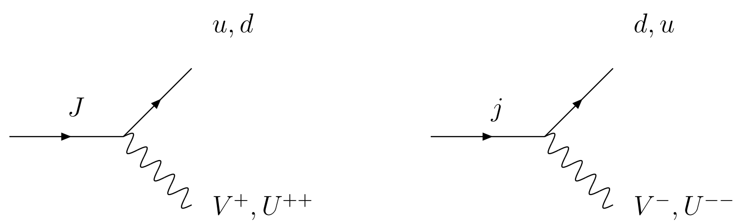

Figure 7: J and j decay in ordinary quarks and bileptons that will decay too.

In the MSSM the charginos, neutralinos, gluinos and squarks pair production leads to a trilepton signature too. The trilepton final states that could arise from the decay of charginos $\tilde{\chi}_1^\pm$ and neutralinos $\tilde{\chi}_2$. For the reaction $\bar{q}q \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0$, where $\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 l^\pm \nu_l$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 l^+ l^-$, and $\tilde{\chi}_1^0$ is the LSP. The ν_l and two LSPs do not interact and manifest themselves as missing energy. The resulting final states is three isolated charged leptons plus missing energy. While the squarks $\tilde{q}$ and the gluinos $\tilde{g}$ have the following interactions $\bar{\tilde{q}}\tilde{q}$, $\tilde{q}\tilde{g}$ and $\tilde{g}\tilde{g}$ and the decays of squarks and gluinos are $\tilde{q} \to q\tilde{\chi}_1^0$ and $\tilde{g} \to \bar{q}q\tilde{\chi}_1^0$ [20, 21, 22].

To finish this analyses from interactions of gauge bosons, we can study $\mathcal{L}_{dc}$ given at Eq.(125) at Appendix A. From this lagrangian we can derive

Case number	Mass relation	decay mode
1	$M_U > M_J$, $M_U > M_j$	$\bar{J}d$, $\bar{u}d$, $l^- l^-$
2	$M_U < M_J$, $M_U > M_j$	$\bar{J}d$, $l^- l^-$
3	$M_U > M_J$, $M_U < M_j$	$\bar{u}d$, $l^- l^-$
4	$M_U < M_J$, $M_U < M_j$	$l^- l^-$

Table 1: All possibles decays to the U boson.

$\underline{l^+l^+d}\ \underbrace{l^-l^-\bar{d}d}$	$\underline{l^+\bar{\nu}u}\ \underbrace{l^-l^-\bar{d}d}$
$\underline{l^+l^+d}\ \underbrace{l^-\nu\bar{u}d}$	$\underline{l^+\bar{\nu}u}\ \underbrace{l^-\nu\bar{u}d}$
$\underline{l^+l^+d}\ \underbrace{l^-l^-\bar{u}u}$	$\underline{l^+\bar{\nu}u}\ \underbrace{l^-l^-\bar{u}u}$
$\underline{l^+l^+d}\ \underbrace{l^-l^-}$	$\underline{l^+\bar{\nu}u}\ \underbrace{l^-l^-}$

Table 2: States coming from JU^{--} decay.

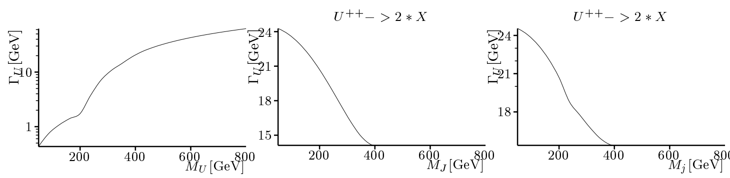

Figure 8: U decay width as function of M_U(left), M_J(center) and M_j(right)

the following Feynman rules given the trilinear and quartic coupling. On this case we get the same results presented [32, 33], given at Tab(3). Here the following notations were used

$$\begin{aligned} S_{\mu\nu,\alpha\beta} &\equiv g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - 2g_{\mu\nu}g_{\alpha\beta}, \\ V_{\mu\nu\alpha\beta} &\equiv g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta}, \\ U_{\mu\beta\nu\alpha} &\equiv g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta}. \end{aligned} \tag{33}$$

4 R-Parity

The R-symmetry was introduced in 1975 by A. Salam and J. Strathdee [34] and in an independent way by P. Fayet [15] to avoid the interactions that violate either lepton number or baryon number. There is very nice review about this subject in Refs.[35, 36]. More precisely, R-parity (which keeps particles invariant, and changes the sign of sparticles) can be written as

$$R = (-1)^{3(B-L)+2S} \tag{34}$$

where S is the spin of the particle.

We said above that only the total lepton number, L, remains a global quantum number (or equivalently we can define $\mathcal{F} = B + L$ as the global conserved quantum number where B is the baryonic number [7]). However,

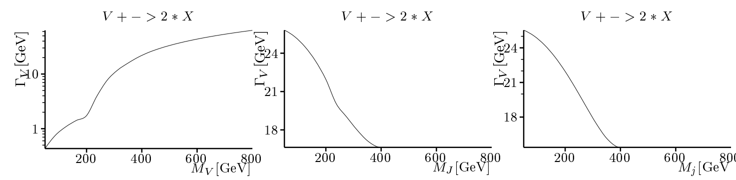

Figure 9: V decay width as function of M_U(left), M_J(center) and M_j(right)

if we assume the global $U(1)_{\mathcal{F}}$ symmetry, it allows us to introduce the R-conserving symmetry, defined as $R = (-1)^{3\mathcal{F}+2S}$. The $\mathcal{F}$ number attribution is

$$\begin{aligned} \mathcal{F}(U^{--}) = \mathcal{F}(V^{-}) = -\mathcal{F}(J_1) = \mathcal{F}(J_{2,3}) = \mathcal{F}(\rho^{--}) \\ = \mathcal{F}(\chi^{--}) = \mathcal{F}(\chi^{-}) = \mathcal{F}(\eta_2^{-}) = 2, \end{aligned} \tag{35}$$

with $\mathcal{F} = 0$ for the other Higgs scalar, while for leptons and the known quarks $\mathcal{F}$ coincides with the total lepton and baryon numbers, respectively.

Choosing the following R-charges

$$\begin{aligned} n_\eta &= n_{\rho'} = -1, \;\; n_\rho = n_{\eta'} = 1, \;\; n_\chi = n_{\chi'} = 0, \\ n_L &= n_{Q_i} = n_{d_i} = 1/2, \;\; n_{J_i} = -1/2, \;\; n_u = -3/2, \end{aligned} \tag{36}$$

it is easy to see that all the fields η, η', χ, χ', ρ, ρ', L, Q_i, u, d and J_i have R-charge equal to one, while their superpartners have opposite R-charge similar to that in the MSSM. The terms which satisfy the defined above symmetry (36) the term allowed by this R-charges in our superpotential, given at Eq.(11), are given by

$$\begin{aligned} W &= \mu_\eta \hat{\eta}\hat{\eta}' + \mu_\rho \hat{\rho}\hat{\rho}' + \mu_\chi \hat{\chi}\hat{\chi}' + \lambda_{2ab}\epsilon \hat{L}_{aL}\hat{L}_{bL}\hat{\eta} + f_1 \epsilon \hat{\rho}\hat{\chi}\hat{\eta} + f_1' \epsilon \hat{\rho}'\hat{\chi}'\hat{\eta}' \\ &+ \kappa_{1\alpha i}\hat{Q}_{\alpha L}\hat{\rho}\hat{u}^c_{iL} + \kappa_{2\alpha i}\hat{Q}_{\alpha L}\hat{\eta}\hat{d}^c_{iL} + \kappa_{3\alpha\beta}\hat{Q}_{\alpha L}\hat{\chi}\hat{j}^c_{\beta L} + \kappa_{4\alpha a i}\hat{Q}_{\alpha L}\hat{L}_{aL}\hat{d}^c_{iL} + \kappa_{5i}\hat{Q}_{3L}\hat{\eta}'\hat{u}^c_{iL} \\ &+ \kappa_{6i}\hat{Q}_{3L}\hat{\rho}'\hat{d}^c_{iL} + \kappa_7 \hat{Q}_{3L}\hat{\chi}'\hat{J}^c_L. \end{aligned} \tag{37}$$

In this case only the quarks get masses. However not all of the leptons get mass. This is because the Yukawa coupling λ_{2ab} is only non-zero when it is antisymmetric in the generation indices (a, b). In the usual 331 model to generate the charged lepton masses we introduce an antisextet, as we don't introduce this scalar in our model the charged leptons are massless in this case. The neutrinos are also massless.

However, if we want to allow neutrinos to get their masses and at the same time avoid the fast nucleon decay we can choose the following R-charges

$$\begin{aligned} n_L &= n_\eta = n_{\eta'} = n_\rho = n_{\rho'} = -n_\chi = n_{\chi'} = 0, \\ n_{Q_i} &= 1,\ \ n_u = n_{d_i} = n_{J_i} = -1. \end{aligned} \tag{38}$$

In this case, the terms allow in our superpotential are

$$\begin{aligned} W &= \mu_{0a}\hat{L}_{aL}\hat{\eta}' + \mu_\eta\hat{\eta}\hat{\eta}' + \mu_\rho\hat{\rho}\hat{\rho}' + \mu_\chi\hat{\chi}\hat{\chi}' + \lambda_{1abc}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{L}_{cL} + \lambda_{2ab}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{\eta} \\ &+ \lambda_{3a}\epsilon\hat{L}_{aL}\hat{\chi}\hat{\rho} + f_1\epsilon\hat{\rho}\hat{\chi}\hat{\eta} + f_1'\epsilon\hat{\rho}'\hat{\chi}'\hat{\eta}' + \kappa_{1\alpha i}\hat{Q}_{\alpha L}\hat{\rho}\hat{u}^c_{iL} + \kappa_{2\alpha i}\hat{Q}_{\alpha L}\hat{\eta}\hat{d}^c_{iL} \\ &+ \kappa_{3\alpha\beta}\hat{Q}_{\alpha L}\hat{\chi}\hat{j}^c_{\beta L} + \kappa_{4\alpha a i}\hat{Q}_{\alpha L}\hat{L}_{aL}\hat{d}^c_{iL} + \kappa_{5i}\hat{Q}_{3L}\hat{\eta}'\hat{u}^c_{iL} + \kappa_{6i}\hat{Q}_{3L}\hat{\rho}'\hat{d}^c_{iL} \\ &+ \kappa_7\hat{Q}_{3L}\hat{\chi}'\hat{J}^c_L. \end{aligned} \tag{39}$$

In our superpotential, we can generate mass to neutrinos, as we will show in the next section, and we get that the nucleon is stable at tree-level [41]. However it is not enough to forbid the dangerous processes of nucleon decay but also forbid the neutron-antineutron oscillation, see Refs. [21, 22, 35, 36, 37].

The last term in this superpotential induce the following nice process[21, 22, 35]

1. Double Beta Decay without Neutrinos
2. New contributions to the Neutrals $K\bar{K}$ and also $B\bar{B}$ Systems;
3. An additional contribution to the muon decay;
4. Charged Current Universality in π and τ decays;
5. Charged Current Universality in the Quark Sector;
6. Leptonic Decays of Heavy Quarks Hadrons such as $D^+ \to \overline{K^0} l^+_i \nu_i$;
7. Rare Leptonic Decays of Mesons like $K^+ \to \pi^+\nu\bar{\nu}$,
8. Hadronic B Meson Decay Asymmetries.

it also give the following direct decays of the lightest neutralinos

$$\begin{aligned} \tilde{\chi}^0_1 &\to l^+_i\bar{u}_j d_k,\ \ \tilde{\chi}^0_1 \to l^-_i u_j\bar{d}_k, \\ \tilde{\chi}^0_1 &\to \bar{\nu}_i\bar{d}_j d_k,\ \ \tilde{\chi}^0_1 \to \nu_i d_j\bar{d}_k, \end{aligned} \tag{40}$$

and for lightest charginos

$$
\begin{aligned}
\tilde{\chi}_1^+ &\rightarrow l_i^+ \bar{d}_j d_k, \;\; \tilde{\chi}_1^+ \rightarrow l_i^+ \bar{u}_j u_k, \\
\tilde{\chi}_1^+ &\rightarrow \bar{\nu}_i \bar{d}_j u_k, \;\; \tilde{\chi}_1^+ \rightarrow \nu_i u_j \bar{d}_k.
\end{aligned} \tag{41}
$$

These very nice aspects also happen in SUSY331RN and in SUSYECO331 models [38].

5 Chargino and Neutralino Production

However, on this model we have doubly charged vector bosons and scalars, respectivelly. This means that in some supersymmetric extensions of these kind of models we will have double charged charginos [9, 39, 40]. On this model the charginos can decay in the following way

$$
\begin{aligned}
\tilde{\chi}^{++} &\rightarrow \tilde{l}^+ l^+, \\
\tilde{\chi}^{+} &\rightarrow \tilde{\nu} l^+, \\
\tilde{\chi}^{0} &\rightarrow \tilde{\nu} \nu.
\end{aligned} \tag{42}
$$

Take this information into account we can say that

$$
e^- e^- \rightarrow \tilde{\chi}^{++} \tilde{\chi}^0 \rightarrow \text{leptons} + \text{mixing energy}. \tag{43}
$$

Again another interesting signal that can be measured at the International Linear Collider (ILC).

In a previous work [9, 39, 40] we have have calculated the total cross section to the reactions

$$
\begin{aligned}
e^- e^- &\rightarrow \tilde{\chi}^- \tilde{\chi}^-, \\
e^- e^- &\rightarrow \tilde{\chi}^{--} \tilde{\chi}^0,
\end{aligned} \tag{44}
$$

we know that the ILC will start to run with $\sqrt{s} = 0,5\text{TeV}$ and therefore this detector can detect them if they really exist in nature.

Naturally these particles can also be detected at LHC through the processes

$$
\begin{aligned}
pp &\rightarrow \tilde{\chi}^+ \tilde{\chi}^+, \\
pp &\rightarrow \tilde{\chi}^{++} \tilde{\chi}^0,
\end{aligned} \tag{45}
$$

and on this case we thinks it will be intersting to study these process to the LHC.

6 Mass spectrum

Here in this article, we want to present the mass spectrum of the Minimal Supersymmetric 3-3-1 model. We will present first the results in the fermion's sector, then in the boson's sector.

6.1 Leptons masses

In a previous work we have shown that, in the MSUSY331, we don't need to use the antisextet to generate the masses to the leptons.

Let us first considered the charged lepton masses. Denoting

$$\begin{array}{c} \phi^{+}=(e^{c},\mu^{c},\tau^{c},-i\lambda^{+}_{W},-i\lambda^{+}_{V},\tilde{\eta}^{\prime +}_{1},\tilde{\eta}^{+}_{2},\tilde{\rho}^{+},\tilde{\chi}^{\prime +})^{T}, \\ \phi^{-}=(e,\mu,\tau,-i\lambda^{-}_{W},-i\lambda^{-}_{V},\tilde{\eta}^{-}_{1},\tilde{\eta}^{\prime -}_{2},\tilde{\rho}^{\prime -},\tilde{\chi}^{-})^{T}, \end{array} \tag{46}$$

where all the fermionic fields are still Weyl spinors, we can also, as before, define $\Psi^{\pm}=(\phi^{+}\phi^{-})^{T}$, and the mass term $-(1/2)[\Psi^{\pm T}Y^{\pm}\Psi^{\pm}+H.c.]$ where $Y^{\pm}$ is given by:

$$Y^{\pm}=\left(\begin{array}{cc} 0 & X^{T} \\ X & 0 \end{array}\right), \tag{47}$$

with

$$X=\left(\begin{array}{ccccccccc}
0 & -\frac{\lambda_{2e\mu}}{3}v & -\frac{\lambda_{2e\tau}}{3}v & 0 & 0 & -\frac{\mu_{0e}}{2} & 0 & -\frac{\lambda_{3e}}{3}w & 0 \\
\frac{\lambda_{2e\mu}}{3}v & 0 & -\frac{\lambda_{2\mu\tau}}{3}v & 0 & 0 & -\frac{\mu_{0\mu}}{2} & 0 & -\frac{\lambda_{3\mu}}{3}w & 0 \\
\frac{\lambda_{2e\tau}}{3}v & \frac{\lambda_{2\mu\tau}}{3}v & 0 & 0 & 0 & -\frac{\mu_{0\tau}}{2} & 0 & -\frac{\lambda_{3\tau}}{3}w & 0 \\
0 & 0 & 0 & m_{\lambda} & 0 & -gv' & 0 & gu & 0 \\
0 & 0 & 0 & 0 & m_{\lambda} & 0 & gv & 0 & -gw' \\
0 & 0 & 0 & gv & 0 & -\frac{\mu_{\eta}}{2} & 0 & \frac{f_1 w}{3} & 0 \\
-\frac{\mu_{0e}}{2} & -\frac{\mu_{0\mu}}{2} & -\frac{\mu_{0\tau}}{2} & 0 & -gv' & 0 & -\frac{\mu_{\eta}}{2} & 0 & -\frac{f'_1 u'}{3} \\
0 & 0 & 0 & -gu' & 0 & \frac{f'_1 w'}{3} & 0 & -\frac{\mu_{\rho}}{2} & 0 \\
-\frac{\lambda_{3e}}{3}u & -\frac{\lambda_{3\mu}}{3}u & -\frac{\lambda_{3\tau}}{3}u & 0 & gw & 0 & -\frac{f_1 u}{3} & 0 & -\frac{\mu_{\chi}}{2}
\end{array}\right), \tag{48}$$

where we have defined v,v',u,u',w and w' at Eq.(20).

The chargino mass matrix $Y^{\pm}$ is diagonalized using two unitary matrices, D and E, defined by

$$\tilde{\chi}^{+}_{i}=D_{ij}\Psi^{+}_{j},\ \ \tilde{\chi}^{-}_{i}=E_{ij}\Psi^{-}_{j},\ \ i,j=1,\cdots,9, \tag{49}$$

(D and E sometimes are denoted, in non-supersymmetric theories, by U^l_R and U^l_L, respectively). Then we can write the diagonal mass matrix as

$$M_{SCM} = E^* X D^{-1}. \tag{50}$$

To determine E and D, we note that

$$M^2_{SCM} = D X^T \cdot X D^{-1} = E^* X \cdot X^T (E^*)^{-1}, \tag{51}$$

and define the following Dirac spinors:

$$\Psi(\tilde{\chi}^+_i) = \left(\begin{array}{cc} \tilde{\chi}^+_i & \bar{\tilde{\chi}}^-_i \end{array} \right)^T, \;\; \Psi^c(\tilde{\chi}^-_i) = \left(\begin{array}{cc} \tilde{\chi}^-_i & \bar{\tilde{\chi}}^+_i \end{array} \right)^T, \tag{52}$$

where $\tilde{\chi}^+_i$ is the particle and $\tilde{\chi}^-_i$ is the anti-particle.

We have obtained the following masses (in GeV) for the charged sector:

$$\begin{aligned} m_{\tilde{\chi}^\pm_9} &= 3186.05, \;\; m_{\tilde{\chi}^\pm_8} = 3001.12, \;\; m_{\tilde{\chi}^\pm_7} = 584.85, \\ m_{\tilde{\chi}^\pm_6} &= 282.30, \;\; m_{\tilde{\chi}^\pm_5} = 204.55, \;\; m_{\tilde{\chi}^\pm_4} = 149.41, \end{aligned} \tag{53}$$

and the masses for the usual leptons (in GeV) $m_e = 0$, $m_\mu = 0.1052$ and $m_\tau = 1.777$.

From the first processes given at Eqs.(44,45) we can detect at ILC or LHC several charginos (since $\tilde{\chi}^\pm_1$ until at least $\tilde{\chi}^\pm_7$) of this model.

These values have been obtained by using the following values for the dimensionless parameters

$$\begin{aligned} &\lambda_{2e\mu} = 0.001, \;\; \lambda_{2e\tau} = 0.001, \;\; \lambda_{2\mu\tau} = 0.393, \\ &\qquad \lambda_{3e} = 0.0001, \;\; \lambda_{3\mu} = 1.0, \;\; \lambda_{3\tau} = 1.0, \end{aligned} \tag{54}$$

$$f_1 = 0.254, \;\; f^\prime_1 = 1.0, \tag{55}$$

and for the mass dimension parameters (in GeV) we have used:

$$\mu_{0e} = \mu_{0\mu} = 0.0, \quad \mu_{0\tau} = 10^{-6}, \tag{56}$$

$$\mu_\eta = 300, \;\; \mu_\rho = 500, \;\; \mu_\chi = 700, \;\; m_\lambda = 3000. \tag{57}$$

We also use the constraint $V^2_\eta + V^2_\rho = (246 \text{ GeV})^2$ coming from M_W, where, we have defined $V^2_\eta = v^2_\eta + v^{\prime 2}_\eta$ and $V^2_\rho = v^2_\rho + v^{\prime 2}_\rho$. Assuming that

$$v_\eta = 20 GeV, \;\; v^\prime_\eta = v^\prime_\rho = 1\text{G}eV, \text{a}nd \;\; 2v_\chi = v^\prime_\chi = 2\text{T}eV, \tag{58}$$

the value of v_ρ is fixed by the constraint above.

Notice, from Eq. (53), that the electron is massless at the tree level. This is a result of the structure of the mass matrix in Eq. (48) and there is not a symmetry that protects the electron to get a mass by loop corrections. We obtain that the dominant contribution to the electron mass is, up to logarithmic corrections,

$$m_e \propto \lambda'_{\alpha ei}\lambda'_{\alpha' ej} V_j^2 V_b^2 (v_\chi^2 + v_{\chi'}^2)\, \frac{m_{j_\alpha}}{9m_{\tilde{b}}^2}, \tag{59}$$

and with all the indices fixed, V_j denotes mixing matrix elements in the two dimension $j_{1,2}$ space, V_b means the same but in the d-like squark sector. We obtain $m_e = 0.0005$ GeV if v_χ and v'_χ have the values already giving above.

6.2 Neutralinos

Like in the case of the charged sector, the neutral lepton masses are given by the mixing among neutrinos, gauginos and higgsinos. The mass term in the basis

$$\Psi^0 = \left(\nu_e \nu_\mu \nu_\tau - i\lambda_A^3 - i\lambda_A^8 - i\lambda_B \tilde{\eta}^0 \tilde{\eta}'^0 \tilde{\rho}^0 \tilde{\rho}'^0 \tilde{\chi}^0 \tilde{\chi}'^0\right)^T, \tag{60}$$

is given by $-(1/2)[(\Psi^0)^T Y^0 \Psi^0 + H.c.]$ where

$$Y^0 = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_{0e}}{2} & \frac{\lambda_{3e}}{3}w & 0 & \frac{\lambda_{3e}}{3}u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_{0\mu}}{2} & \frac{\lambda_{3\mu}}{3}w & 0 & \frac{\lambda_{3\mu}}{3}u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_{0\tau}}{2} & \frac{\lambda_{3\tau}}{3}w & 0 & \frac{\lambda_{3\tau}}{3}u & 0 \\
0 & 0 & 0 & m_\lambda & 0 & 0 & \frac{gv}{\sqrt{2}} & -\frac{gv'}{\sqrt{2}} & -\frac{gu}{\sqrt{2}} & \frac{gu'}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & m_\lambda & 0 & \frac{gv}{\sqrt{6}} & -\frac{gv'}{\sqrt{6}} & \frac{gu}{\sqrt{6}} & -\frac{gu'}{\sqrt{6}} & -\frac{2}{\sqrt{6}}gw & \frac{2}{\sqrt{6}}gw' \\
0 & 0 & 0 & 0 & 0 & m' & 0 & 0 & \frac{g'u}{\sqrt{2}} & -\frac{g'u'}{\sqrt{2}} & -\frac{g'w}{\sqrt{2}} & \frac{g'w'}{\sqrt{2}} \\
0 & 0 & 0 & \frac{gv}{\sqrt{2}} & \frac{gv}{\sqrt{6}} & 0 & 0 & -\frac{\mu_\eta}{2} & -\frac{f_1 w}{3} & 0 & \frac{f_1 u}{3} & 0 \\
-\frac{\mu_{0e}}{2} & -\frac{\mu_{0\mu}}{2} & -\frac{\mu_{0\tau}}{2} & -\frac{gv'}{\sqrt{2}} & -\frac{gv'}{\sqrt{6}} & 0 & -\frac{\mu_\eta}{2} & 0 & 0 & -\frac{f'_1 w'}{3} & 0 & \frac{f'_1 u'}{3} \\
\frac{\lambda_{3e}}{3}w & \frac{\lambda_{3\mu}}{3}w & \frac{\lambda_{3\tau}}{3}w & -\frac{gu}{\sqrt{2}} & \frac{gu}{\sqrt{6}} & \frac{g'u}{\sqrt{2}} & -\frac{f_1 w}{3} & 0 & 0 & -\frac{\mu_\rho}{2} & -\frac{f_1 v}{3} & 0 \\
0 & 0 & 0 & \frac{gu'}{\sqrt{2}} & -\frac{gu'}{\sqrt{6}} & -\frac{g'u'}{\sqrt{2}} & 0 & -\frac{f'_1 w'}{3} & -\frac{\mu_\rho}{2} & 0 & 0 & -\frac{f'_1 v'}{3} \\
\frac{\lambda_{3e}}{3}u & \frac{\lambda_{3\mu}}{3}u & \frac{\lambda_{3\tau}}{3}u & 0 & -\frac{2}{\sqrt{6}}gw & -\frac{g'w}{\sqrt{2}} & \frac{f_1 u}{3} & 0 & -\frac{f_1 v}{3} & 0 & 0 & -\frac{\mu_\chi}{2} \\
0 & 0 & 0 & 0 & \frac{2}{\sqrt{6}}gw' & \frac{g'w'}{\sqrt{2}} & 0 & \frac{f'_1 u'}{3} & 0 & -\frac{f'_1 v'}{3} & -\frac{\mu_\chi}{2} & 0
\end{pmatrix}. \tag{61}$$

All parameters in Eq. (61), but m', are defined in Eqs. (20), (54) and (57); g and g' denote the gauge coupling constant of $SU(3)_L$ and $U(1)_N$, respectively.

The neutralino mass matrix is diagonalized by a 12×12 rotation unitary matrix N, satisfying

$$M_{NMD} = N^* Y^0 N^{-1}, \tag{62}$$

and the mass eigenstates are

$$\tilde{\chi}^0_i \;=\; N_{ij}\Psi^0_j, \;\; j=1,\cdots,12. \tag{63}$$

We can define the following Majorana spinor to represent the mass eigenstates

$$\Psi(\tilde{\chi}^0_i) \;=\; \left(\begin{array}{cc} \tilde{\chi}^0_i & \bar{\tilde{\chi}}^0_i \end{array} \right)^T. \tag{64}$$

As above the subindices a, b, c run over the lepton generations e, μ, τ.

With the mass matrix in Eq. (61), at the tree level we obtain the eigenvalues (in GeV),

$$\begin{array}{c} m_{\tilde{\chi}^0_{12}} = -4162.22, \;\; m_{\tilde{\chi}^0_{11}} = 3260.48, \;\; m_{\tilde{\chi}^0_{10}} = 3001.11, \\ m_{\tilde{\chi}^0_{9}} = 585.19, \;\; m_{\tilde{\chi}^0_{8}} = -585.19, \;\; m_{\tilde{\chi}^0_{7}} = 453.22, \\ m_{\tilde{\chi}^0_{6}} = -344.14, \;\; m_{\tilde{\chi}^0_{5}} = 283.14, \;\; m_{\tilde{\chi}^0_{4}} = -272.0, \end{array} \tag{65}$$

and for the three neutrinos we obtain (in eV)

$$m_{\tilde{\chi}^0_1} = 0, \; m_{\tilde{\chi}^0_2} \approx -0.01, \; m_{\tilde{\chi}^0_3} \approx 1.44. \tag{66}$$

We have got the values in Eqs. (65) and (66) by choosing, besides the parameters in Eqs. (54) and (57), $m' = -3780.4159$ GeV. Notice that the coupling constant g' and the parameter m' appear only in the mass matrix of the neutralinos, all the other parameters in Eq. (61) have already been fixed by the charged sector, see Eq. (48), (54) and (57). The neutrino masses in Eq. (66) are of the order of magnitude for LSND and solar neutrino data.

6.3 Double Charged Charginos

Introducing the notation

$$\psi^{++} = \left(\begin{array}{ccc} -i\lambda^{++}_U & \tilde{\rho}^{++} & \tilde{\chi}'^{++} \end{array} \right)^t, \;\; \psi^{--} = \left(\begin{array}{ccc} -i\lambda^{--}_U & \tilde{\rho}'^{--} & \tilde{\chi}^{--} \end{array} \right)^t,$$

and

$$\Psi^{\pm\pm} = \left(\begin{array}{cc} \psi^{++} & \psi^{--} \end{array} \right)^t, \tag{67}$$

we can write the following equation [9]

$$\mathcal{L}^{\rm double}_{\rm mass} \;=\; -\frac{1}{2}\left(\Psi^{\pm\pm}\right)^t Y^{\pm\pm}\Psi^{\pm\pm} + hc, \tag{68}$$

where

$$Y^{\pm\pm} = \begin{pmatrix} 0 & T^t \\ T & 0 \end{pmatrix}, \tag{69}$$

with

$$T = \begin{pmatrix} -m_\lambda & -gu & gw' & \frac{gz}{\sqrt{2}} & -\frac{gz'}{\sqrt{2}} \\ gu' & \frac{\mu_\rho}{2} & -\left(\frac{f_1' v'}{3} - \sqrt{2}\frac{f_3'}{3}z'\right) & 0 & \frac{f_3'}{3}w' \\ -gw & -\left(\frac{f_1 v}{3} - \sqrt{2}\frac{f_3}{3}z\right) & \frac{\mu_\chi}{2} & \frac{f_3}{3}u & 0 \\ -\frac{gz'}{\sqrt{2}} & 0 & \frac{f_3'}{3}u' & \frac{\mu_S}{2} & 0 \\ \frac{gz}{\sqrt{2}} & \frac{f_3}{3}w & 0 & 0 & \frac{\mu_S}{2} \end{pmatrix}. \tag{70}$$

The matrix $Y^{\pm\pm}$ in Eq.(69) satisfy the following relation

$$\det(Y^{\pm\pm} - \lambda I) = \det\left[\begin{pmatrix} -\lambda & T^t \\ T & -\lambda \end{pmatrix}\right] = \det(\lambda^2 - T^t \cdot T), \tag{71}$$

so we only have to calculate $T^t \cdot T$ to obtain the eigenvalues. Since $T^t \cdot T$ is a symmetric matrix, λ^2 must be real, and positive because $Y^{\pm\pm}$ is also symmetric.

The double chargino mass matrix is diagonalized using two rotation matrices, A and B, defined by

$$\tilde{\chi}_i^{++} = A_{ij}\Psi_j^{++}, \;\; \tilde{\chi}_i^{--} = B_{ij}\Psi_j^{--}, \;\; i, j = 1, \cdots, 5. \tag{72}$$

where A and B are unitary matrices such that

$$M_{DCC} = B^* T A^{-1}, \tag{73}$$

the matrix T is defined in Eq.(70). To determine A and B, we note that

$$M_{DCC}^2 = A T^t \cdot T A^{-1} = B^* T \cdot T^t (B^*)^{-1}, \tag{74}$$

which means that A diagonalizes $T^t \cdot T$ while B diagonalizes $T \cdot T^t$. It means

$$\mathrm{diag}(m_{\tilde{\chi}^{\pm\pm}}) \equiv [B^* T A^{-1}]_{ij} = m_{\tilde{\chi}_i^{\pm\pm}} \delta_{ij}. \tag{75}$$

Performing the diagonalization we get the following numerical results in *GeV*

$$m_{\tilde{\chi}_1^{\pm\pm}} = 194.4, \;\; m_{\tilde{\chi}_2^{\pm\pm}} = 343.3, \;\; m_{\tilde{\chi}_3^{\pm\pm}} = 452.2, \;\; m_{\tilde{\chi}_4^{\pm\pm}} = 652.1, \;\; m_{\tilde{\chi}_5^{\pm\pm}} = 3187.$$

Using this equation togheter Eqs(65,66) and considerating the second processes given at Eqs.(44,45) we can detect at ILC or LHC several double charginos (since $\tilde{\chi}^{\pm\pm}_1$ until at least $\tilde{\chi}^{\pm\pm}_4$) and neutralinos (since $\tilde{\chi}^0_1$ until at least $\tilde{\chi}^0_9$) of this model.

We define the following Dirac spinors to represent the mass eigenstates:

$$\Psi(\tilde{\chi}^{++}_i) = \left(\begin{array}{cc} \tilde{\chi}^{++}_i & \bar{\tilde{\chi}}^{--}_i \end{array} \right)^t, \;\; \Psi^c(\tilde{\chi}^{--}_i) = \left(\begin{array}{cc} \tilde{\chi}^{--}_i & \bar{\tilde{\chi}}^{++}_i \end{array} \right)^t, \tag{76}$$

where $\tilde{\chi}^{++}_i$ is the particle and $\tilde{\chi}^{--}_i$ is the anti-particle, (we are using the same notation as in [14]).

6.4 Quarks masses

Let us first considered the u-quarks type.There are interactions like

$$-\left[\frac{\kappa_{1i}}{3} \left(Q_3 \eta^\prime u^c_i + \bar{Q}_3 \overline{\eta}^\prime \bar{u}^c_i \right) + \frac{\kappa_{5\alpha i}}{3} \left(Q_\alpha \rho u^c_i + \bar{Q}_\alpha \overline{\rho} \bar{u}^c_i \right) \right], \tag{77}$$

which imply a general mixing in the u-quark sector. Denoting

$$\psi^+_u = \left(\begin{array}{ccc} u_1 & u_2 & u_3 \end{array} \right)^T, \;\; \psi^-_u = \left(\begin{array}{ccc} u^c_1 & u^c_2 & u^c_3 \end{array} \right)^T, \tag{78}$$

where all the u-quarks fields are still Weyl spinors, we can also, define $\Psi^\pm_u = (\psi^+_u \psi^-_u)^T$. We can define the mass term $-(1/2)[\Psi^{\pm T}_u Y^\pm_u \Psi^\pm_u + H.c.]$ where $Y^\pm_u$ is given by:

$$Y^\pm_u = \left(\begin{array}{cc} 0 & X^T_u \\ X_u & 0 \end{array} \right), \tag{79}$$

with

$$X_u = \frac{1}{3} \left(\begin{array}{ccc} \kappa_{511} u & \kappa_{521} u & \kappa_{11} v^\prime \\ \kappa_{512} u & \kappa_{522} u & \kappa_{12} v^\prime \\ \kappa_{513} u & \kappa_{523} u & \kappa_{13} v^\prime \end{array} \right), \tag{80}$$

where the VEVs are defined in Eq.(20)

The u-quarks mass matrix $Y^\pm_u$ is diagonalized using two rotation matrices, D and E, defined by

$$u^+_i = D_{ij} \psi^+_{uj}, \;\; u^-_i = E_{ij} \psi^-_{uj}, \;\; i,j = 1,2,3. \tag{81}$$

Then we can write the diagonal matrix (D and E are unitary) as

$$M_u = E^* X_u D^{-1}. \tag{82}$$

To determine D and E, we note that

$$M_u^2 = DX_u^T X_u D^{-1} = E^* X_u X_u^T (E^*)^{-1}, \tag{83}$$

and define the following Dirac spinors

$$\Psi(u^+) = \begin{pmatrix} u^+ & \bar{u}^- \end{pmatrix}^T, \;\; \Psi^c(u^-) = \begin{pmatrix} u^- & \bar{u}^+ \end{pmatrix}^T. \tag{84}$$

To the d-quark type.There are interactions like

$$-\left[\frac{\kappa_{2i}}{3}\left(Q_3\rho' d_i^c + \bar{Q}_3\bar{\rho}'\bar{d}_i^c\right) + \frac{\kappa_{4\alpha i}}{3}\left(Q_\alpha \eta d_i^c + \bar{Q}_\alpha \bar{\eta}\bar{d}_i^c\right)\right], \tag{85}$$

which imply a general mixing in the d-quark sector. Denoting

$$\psi_d^+ = \begin{pmatrix} d_1^c & d_2^c & d_3^c \end{pmatrix}^T, \;\; \psi_d^- = \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix}^T, \tag{86}$$

where all the d-quarks fields are still Weyl spinors, we can also, define $\Psi_d^\pm = (\psi_d^+ \psi_d^-)^T$. We can define the mass term $-(1/2)[\Psi_d^{\pm T} Y_d^\pm \Psi_d^\pm + H.c.]$ where $Y_d^\pm$ is given by:

$$Y_d^\pm = \begin{pmatrix} 0 & X_d^T \\ X_d & 0 \end{pmatrix}, \tag{87}$$

with

$$X_d = \frac{1}{3}\begin{pmatrix} \kappa_{411}v & \kappa_{412}v & \kappa_{413}v \\ \kappa_{421}v & \kappa_{422}v & \kappa_{423}v \\ \kappa_{21}u' & \kappa_{22}u' & \kappa_{23}u' \end{pmatrix}, \tag{88}$$

where all the VEVs are defined in Eq.(20)

The d-quarks mass matrix $Y_d^\pm$ is diagonalized using two rotation matrices, F and G, defined by

$$d_i^+ = F_{ij}\psi_{dj}^+, \;\; d_i^- = G_{ij}\psi_{uj}^-, \;\; i,j = 1,2,3. \tag{89}$$

Then we can write the diagonal matrix (F and G are unitary) as

$$M_d = G^* X_d F^{-1}. \tag{90}$$

To determine F and G, we note that

$$M_d^2 = FX_d^T X_d F^{-1} = G^* X_d X_d^T (G^*)^{-1}, \tag{91}$$

and define the following Dirac spinors

$$\Psi(d^{+}) = \begin{pmatrix} d^{+} & \bar{d^{-}} \end{pmatrix}^{T}, \;\; \Psi^{c}(d^{-}) = \begin{pmatrix} d^{-} & \bar{d^{+}} \end{pmatrix}^{T}. \tag{92}$$

In general the Yukawa couplings $\kappa_{1i}, \kappa_{5\alpha i}, \kappa_{2i}$ and $\kappa_{4\alpha i}$ are different of zero and all quarks get their masses as happen in the SM and in the MSSM. However, we know that the quarks s, t are heavier then the quark c, b and the quark u is lighter than the quark d.

We can try to given an reasonable explanation about these mass hierarchy in this model. In order to get this explanation we can suppose that the Yukawa couplings $\mathcal{O}(\kappa_{11}) = \mathcal{O}(\kappa_{12}) = \mathcal{O}(\kappa_{21}) = \mathcal{O}(\kappa_{22})$ are much smaller than the other Yukawa couplings that appear at mass matrices of the usual quarks. This hypothesis means we are going to neglect the mixing between the first and second familly with the first familly.

Under this supposition we can rewrite our mass matrices, given at Eqs.(80,88), in the following way

$$\begin{aligned} X_{u} &\simeq \frac{1}{3} \begin{pmatrix} \kappa_{511} & \kappa_{521} \\ \kappa_{512} & \kappa_{522} \end{pmatrix} u, \\ X_{d} &\simeq \frac{1}{3} \begin{pmatrix} \kappa_{411} & \kappa_{412} \\ \kappa_{421} & \kappa_{422} \end{pmatrix} v \end{aligned} \tag{93}$$

We know from Eq.(58) that $u > v$ then Eq.(93) explain why the quarks of charge $(2e)/3$ are heavier than the quarks of charge $(-e)/3$.

By another way

$$m_{u} = \frac{1}{3} \kappa_{13} v', \;\; m_{d} = \frac{1}{3} \kappa_{23} u', \tag{94}$$

but, from Eq.(58) we notice that $v' \sim u'$ and if $\kappa_{23} > \kappa_{13}$ we can explain why d-quark is a little more heavier than u-quark.

There are another way to try to explain the mass hierarchy between the fermions. If we look from Eqs.(48,80,88), it is easy to note that we can prevent u, d, s and e from picking up tree-level masses. To get this result we need to impose the following $\mathcal{Z}_2'$ symmetry on the Lagrangian

$$\widehat{d}^{c}_{2} \longrightarrow -\widehat{d}^{c}_{2}, \;\; \widehat{d}^{c}_{3} \longrightarrow -\widehat{d}^{c}_{3}, \;\; \widehat{u}^{c}_{3} \longrightarrow -\widehat{u}^{c}_{3}, \;\; \widehat{l}^{c}_{3} \longrightarrow -\widehat{l}^{c}_{3}, \tag{95}$$

the others superfields are even under this symmetry as showed at Ref. [42]. On this case, it was showed that under $\mathcal{Z}_2'$ symmetry the heavy quarks c, b

and t acquire mass at tree level while the light quarks (u, d, s) get their mass at 1-loop level.

There is another intersting possibility to get u, d light. We can introduce a new discrete T' flavor symmetry as done in [43].

6.5 Masses of Exotic quarks

For the J-quark type. There are interactions like

$$-\left[\frac{\kappa_3}{3}\left(Q_3\chi' J^c + \bar{Q}_3\overline{\chi'}\bar{J}^c\right)\right], \tag{96}$$

which imply one diagonalized state with the following mass

$$M_J^{mass} = -\frac{\kappa_3}{3}\omega'\left(JJ^c + \bar{J}\bar{J}^c\right). \tag{97}$$

The another exotic quark j. There are interactions like

$$-\left[\frac{\kappa_{6\alpha\beta}}{3}\left(Q_\alpha\chi j^c_\beta + \bar{Q}_\alpha\overline{\chi}\bar{j}^c_\beta\right)\right], \tag{98}$$

which imply a general mixing in the j-quark sector. Denoting

$$\psi_j^+ = \left(\begin{array}{cc} j_1^c & j_2^c \end{array}\right)^T, \;\; \psi_j^- = \left(\begin{array}{cc} j_1 & j_2 \end{array}\right)^T, \tag{99}$$

where all the j-quarks fields are still Weyl spinors, we can also, define $\Psi_j^\pm = \left(\psi_j^+, \psi_j^-\right)^T$. We can define the mass term $-(1/2)\Psi_j^{\pm T} Y_j^\pm \Psi_j^\pm + H.c.]$ where $Y_j^\pm$ is given by:

$$Y_j^\pm = \left(\begin{array}{cc} 0 & X_j^t \\ X_j & 0 \end{array}\right), \tag{100}$$

with

$$X_j = \frac{1}{3}\left(\begin{array}{cc} \kappa_{611}w & \kappa_{612}w \\ \kappa_{621}w & \kappa_{622}w \end{array}\right), \tag{101}$$

where the values VEVs are defined in Eq.(20).

The j-quarks mass matrix is diagonalized using two rotation matrices, H and I, defined by

$$j^+_\alpha = H_{\alpha\beta}\psi^+_{j_\beta}, \;\; j^-_\alpha = I_{\alpha\beta}\psi^-_{j_\beta}, \;\; \alpha, \beta = 1, 2. \tag{102}$$

Then we can write the diagonal matrix (H and I are unitary) as

$$M_j = I^* X_j H^{-1}. \tag{103}$$

To determine I and H, we note that

$$M_j^2 = H X_j^T X_j H^{-1} = I^* X_j X_j^T (I^*)^{-1}. \tag{104}$$

The masses of physical j are

$$\begin{aligned} M_{j_1} &= \frac{1}{6}\left(\kappa_{611} + \kappa_{622} + \sqrt{(\kappa_{611} - \kappa_{622})^2 + 4\kappa_{612}\kappa_{621}}\right) w, \\ M_{j_2} &= \frac{1}{6}\left(\kappa_{611} + \kappa_{622} - \sqrt{(\kappa_{611} - \kappa_{622})^2 + 4\kappa_{612}\kappa_{621}}\right) w. \end{aligned} \tag{105}$$

Notice that if κ_{612} is zero we get that

$$M_{j_1} = \frac{\kappa_{611}}{3} w, \quad \text{and} \quad M_{j_2} = \frac{\kappa_{622}}{3} w, \tag{106}$$

therefore κ_{612} and κ_{621} can be both zero that both j_1 and j_2 are massive.

If we consider that $\mathcal{O}(\kappa_3)$, $\mathcal{O}(\kappa_{611})$ and $\mathcal{O}(\kappa_{622})$ are the same order, for example $\mathcal{O} \simeq 10^{-2}$ for example, it means from Eqs.(58,97,106) that J-quark is heavier than $j_{1,2}$ quarks and their masses are in TeV scale.

We define the following Dirac spinors

$$\Psi(j^+) = \begin{pmatrix} j^+ & \bar{j^-} \end{pmatrix}^T, \quad \Psi^c(j^-) = \begin{pmatrix} j^- & \bar{j^+} \end{pmatrix}^T. \tag{107}$$

6.6 The masses of Gluinos

It is well known gluinos are the supersymmetric partners of the gluons. Therefore gluinos, as in the MSSM, are the color octet fermions in the model and due the fact that the $SU(3)_c$ group is unbroken, it means the gluinos can not mix with any others particles in the model, then they are already mass eigenstates.

Their mass, is one of the soft parameter that break SUSY, can be written as

$$\mathcal{L}^{\text{gluino}}_{\text{mass}} = \frac{m_{\tilde{g}}}{2} \bar{\tilde{g}} \tilde{g} \tag{108}$$

so that its mass at tree level is $m_{\tilde{g}} = |m_{\lambda_c}|$, as denoted at Eq.(16) and Refs. [8, 9], where

$$\tilde{g}^a = \begin{pmatrix} -\imath\lambda^a_C \\ \imath\overline{\lambda^a_C} \end{pmatrix}, \qquad a = 1, \ldots, 8, \tag{109}$$

is the Majorana four-spinor defining the physical gluinos states.

6.7 Gauge Bosons masses

The gauge mass term is given by $\mathcal{L}^{\rm scalar}_{HHVV}$ which we can divided in $\mathcal{L}^{\rm charged}_{\rm mass}$ and $\mathcal{L}^{\rm neutral}_{\rm mass}$, see Refs. [8, 9].

The neutral gauge boson mass is given by

$$\mathcal{L}^{\rm neutral}_{\rm mass} = \begin{pmatrix} V_{3m} & V_{8m} & V_m \end{pmatrix} M^2 \begin{pmatrix} V_3^m \\ V_8^m \\ V^m \end{pmatrix}, \qquad (110)$$

where

$$M^2 = \frac{g^2}{2}\begin{pmatrix} (v^2+u^2) & \frac{1}{\sqrt{3}}(v^2-u^2) & -2tu^2 \\ \frac{1}{\sqrt{3}}(v^2-u^2) & \frac{1}{3}(v^2+u^2+4w^2) & \frac{2t}{\sqrt{3}}(u^2+2w^2) \\ -2tu^2 & \frac{2t}{\sqrt{3}}(u^2+2w^2) & 4t^2(u^2+w^2) \end{pmatrix}, \qquad (111)$$

with $t = g'/g$.

In the approximation that $w^2 \gg v^2, u^2$, the masses of the neutral gauge bosons are: 0, M^2_Z and $M^2_{Z'}$, and the masses are given by

$$M^2_Z \approx \frac{1}{4}\frac{g^2+4g'^2}{g^2+3g'^2}(v^2_\eta + v^2_\rho + v^2_{\eta'} + v^2_{\rho'}), \;\; M^2_{Z'} \approx \frac{1}{3}(g^2+3g'^2)(v^2_\chi + v^2_{\chi'}). \quad (112)$$

The charged gauge boson mass term, $\mathcal{L}^{\rm neutral}_{\rm mass}$ see Refs. [8, 9], can be written as

$$\mathcal{L}^{\rm charged}_{\rm mass} = M^2_W W^-_m W^{+m} + M^2_V V^-_m V^{+m} + M^2_U U^{--}_m U^{++m}, \qquad (113)$$

where

$$\begin{aligned} M^2_U &= \frac{g^2}{4}(v^2_\rho + v^2_\chi + v^2_{\rho'} + v^2_{\chi'}), \\ M^2_W &= \frac{g^2}{4}(v^2_\eta + v^2_\rho + v^2_{\eta'} + v^2_{\rho'}), \\ M^2_V &= \frac{g^2}{4}(v^2_\eta + v^2_\chi + v^2_{\eta'} + v^2_{\chi'}). \end{aligned} \qquad (114)$$

Comparing Eqs.(114,112) we can conclude

$$M_{Z'} > M_U > M_V > M_Z > M_W. \qquad (115)$$

Using M_W given in Eq.(114) and M_Z in Eq.(112) we get the following relation:

$$\frac{M_Z^2}{M_W^2}=\frac{1+4t^2}{1+3t^2}=\frac{1}{1-\sin^2\theta_W}, \tag{116}$$

therefore we obtain

$$t^2=\frac{\sin^2\theta_W}{1-4\sin^2\theta_W}. \tag{117}$$

We want to mention that the gauge boson sector is exactly the same as in the non-supersymmetric 3-3-1 model.

Using Eq.(58) in Eqs.(112,114) we get the following masses values for the gauge bosons

$$\begin{aligned} M_U &= 734,63\ \text{GeV},\ \ M_V=730,28\ \text{GeV},\ \ M_W=80,40\text{GeV}, \\ M_Z &= 91,3\ \text{GeV},\ \ M_{Z'}=2698,94\ \text{GeV}. \end{aligned} \tag{118}$$

These values satisfy the Eq.(115) and M_W and M_Z are in agreement with the experimental limits. The lower limit in Z' boson is $M_{Z'}>822$ GeV [44] and our mass is in agreement with this experimental limit.

7 Conclusions

In this paper we have presented new R-symmetry for the minimal supersymmetric $\mathrm{SU}(3)_C\otimes\mathrm{SU}(3)_L\otimes\mathrm{U}(1)_X$ model and studied all the spectrum from the fermion's sector and gauged's boson sector of this model. We also show that some of the new state as $\tilde{\chi}^{\pm\pm}_{1\ldots4}$, $\tilde{\chi}^{\pm\pm}_{1\ldots7}$, $\tilde{\chi}^{0}_{1\pm9}$, j_1 and J can be discovered by LHC or same ILC, if they really exist.

The new R-parity not only provides a simple mechanism for the mass generation of the neutrinos but also gives some lepton flavor violating interactions at the tree level. This will play some important phenomenology in our model such as the proton's stability, forbiddance of the neutron-antineutron oscillation and neutrinoless double beta decay.

Acknowledgments

This work was financed by the Brazilian funding agency CNPq, under contract number 309564/2006-9.

A Lagrangian

The goal of this Appendix is to present all terms in the lagrangian of the model, which we have used in this work.

A.1 Lepton Lagrangian

In the $\mathcal{L}_{\text{Lepton}}$ term can be written as [23]

$$\mathcal{L}_{\text{Lépton}} = \mathcal{L}_{llV} + \mathcal{L}_{\tilde{l}\tilde{l}V} + \mathcal{L}_{l\tilde{l}\tilde{V}} + \mathcal{L}_{\tilde{l}\tilde{l}VV} + \mathcal{L}^{leptons}_{cin} + \mathcal{L}^{leptons}_{F} + \mathcal{L}^{leptons}_{D}, \tag{119}$$

where

$$\begin{aligned}
\mathcal{L}_{llV} &= \frac{g}{2}\bar{L}\bar{\sigma}^m\lambda^a L V^a_m; \;\; \mathcal{L}_{\tilde{l}\tilde{l}V} = -\frac{ig}{2}\left[\tilde{L}\lambda^a\partial^m\bar{\tilde{L}} - \bar{\tilde{L}}\lambda^a\partial^m\tilde{L}\right]V^a_m, \\
\mathcal{L}_{l\tilde{l}\tilde{V}} &= -\frac{ig}{\sqrt{2}}(\bar{L}\lambda^a\tilde{L}\bar{\lambda}^a_A - \bar{\tilde{L}}\lambda^a L\lambda^a_A); \;\; \mathcal{L}_{\tilde{l}\tilde{l}VV} = \frac{g^2}{4}V^a_m V^{bm}\bar{\tilde{L}}\lambda^a\lambda^b\tilde{L}, \\
\mathcal{L}^{leptons}_{cin} &= \tilde{L}\partial^2\tilde{L}^* - iL\sigma^m\partial_m\bar{L}; \;\; \partial^2 = \partial^m\partial_m, \\
\mathcal{L}^{leptons}_{F} &= |F_L|^2; \\
\mathcal{L}^{leptons}_{D} &= \frac{g}{2}\bar{\tilde{L}}\lambda^a\tilde{L}D^a.
\end{aligned} \tag{120}$$

A.2 Quark Lagrangian

As above we can write to the quarks

$$\begin{aligned}
\mathcal{L}_{qqV} &= \frac{g_s}{2}(\bar{Q}_i\bar{\sigma}^m\lambda^a Q_i - \bar{u}^c_i\bar{\sigma}^m\lambda^{*a}u^c_i - \bar{d}^c_i\bar{\sigma}^m\lambda^{*a}d^c_i - \bar{J}^c_i\bar{\sigma}^m\lambda^{*a}J^c_i)g^a_m \\
&+ \frac{g}{2}(\bar{Q}_3\bar{\sigma}^m\lambda^a Q_3 - \bar{Q}_\alpha\bar{\sigma}^m\lambda^{*a}Q_\alpha)V^a_m \\
&+ \frac{g'}{2}\left(\frac{2}{3}\bar{Q}_3\bar{\sigma}^m Q_3 - \frac{1}{3}\bar{Q}_\alpha\bar{\sigma}^m Q_\alpha - \frac{2}{3}\bar{u}^c_i\bar{\sigma}^m u^c_i + \frac{1}{3}\bar{d}^c_i\bar{\sigma}^m d^c_i - \frac{5}{3}\bar{J}^c\bar{\sigma}^m J^c + \frac{4}{3}\bar{j}^c_\beta\bar{\sigma}^m j^c_\beta\right)V_m, \\
\mathcal{L}_{\tilde{q}\tilde{q}V} &= \frac{-ig_s}{2}\Big[(\tilde{Q}_i\lambda^a\partial^m\bar{\tilde{Q}}_i - \bar{\tilde{Q}}_i\lambda^a\partial^m\tilde{Q}_i - \tilde{u}^c_i\lambda^{*a}\partial^m\bar{\tilde{u}}^c_i + \bar{\tilde{u}}^c_i\lambda^{*a}\partial^m\tilde{u}^c_i \\
&- \tilde{d}^c_i\lambda^{*a}\partial^m\bar{\tilde{d}}^c_i + \bar{\tilde{d}}^c_i\lambda^{*a}\partial^m\tilde{d}^c_i - \tilde{J}^c_i\lambda^{*a}\partial^m\bar{\tilde{J}}^c_i + \bar{\tilde{J}}^c_i\lambda^{*a}\partial^m\tilde{J}^c_i)g^a_m\Big] \\
&- \frac{ig}{2}(\tilde{Q}_3\lambda^a\partial^m\bar{\tilde{Q}}_3 - \bar{\tilde{Q}}_3\lambda^a\partial^m\tilde{Q}_3 - \tilde{Q}_\alpha\lambda^{*a}\partial^m\bar{\tilde{Q}}_\alpha + \bar{\tilde{Q}}_\alpha\lambda^{*a}\partial^m\tilde{Q}_\alpha)V^a_m
\end{aligned}$$

$$
\begin{aligned}
&- && \frac{ig'}{2}\left[\frac{2}{3}(\tilde{Q}_3\partial^m\bar{\tilde{Q}}_3-\bar{\tilde{Q}}_3\partial^m\tilde{Q}_3)-\frac{1}{3}(\tilde{Q}_\alpha\partial^m\bar{\tilde{Q}}_\alpha-\bar{\tilde{Q}}_\alpha\partial^m\tilde{Q}_\alpha)-\frac{2}{3}(\tilde{u}^c_i\partial^m\bar{\tilde{u}}^c_i-\bar{\tilde{u}}^c_i\partial^m\tilde{u}^c_i)\right.\\
&+ && \left.\frac{1}{3}(\tilde{d}^c_i\partial^m\bar{\tilde{d}}^c_i-\bar{\tilde{d}}^c_i\partial^m\tilde{d}^c_i)-\frac{5}{3}(\tilde{J}^c\partial^m\bar{\tilde{J}}^c-\bar{\tilde{J}}^c\partial^m\tilde{J}^c)+\frac{4}{3}(\tilde{j}^c_\beta\partial^m\bar{\tilde{j}}^c_\beta-\bar{\tilde{j}}^c_\beta\partial^m\tilde{j}^c_\beta)\right]V_m,\\
\mathcal{L}_{q\tilde{q}\tilde{V}} &= && \frac{-ig_s}{\sqrt{2}}\left[(\bar{Q}_i\lambda^a\tilde{Q}_i-\bar{u}^c_i\lambda^{*a}\tilde{u}^c_i-\bar{d}^c_i\lambda^{*a}\tilde{d}^c_i-\bar{J}^c_i\lambda^{*a}\tilde{J}^c_i)\bar{\lambda}^a_c\right.\\
&- && \left.(\bar{\tilde{Q}}_i\lambda^aQ_i-\bar{\tilde{u}}^c_i\lambda^{*a}u^c_i-\bar{\tilde{d}}^c_i\lambda^{*a}d^c_i-\bar{\tilde{J}}^c_i\lambda^{*a}J^c_i)\lambda^a_c\right]\\
&- && \frac{ig}{\sqrt{2}}\left[(\bar{Q}_3\lambda^a\tilde{Q}_3-\bar{Q}_\alpha\lambda^{*a}\tilde{Q}_\alpha)\bar{\lambda}^a_A-(\bar{\tilde{Q}}_3\lambda^aQ_3-\bar{\tilde{Q}}_\alpha\lambda^{*a}Q_\alpha)\lambda^a_A\right]\\
&- && \frac{ig'}{\sqrt{2}}\left[\left(\frac{2}{3}\bar{Q}_3\tilde{Q}_3-\frac{1}{3}\bar{Q}_\alpha\tilde{Q}_\alpha-\frac{2}{3}\bar{u}^c_i\tilde{u}^c_i+\frac{1}{3}\bar{d}^c_i\tilde{d}^c_i-\frac{5}{3}\bar{J}^c\tilde{J}^c+\frac{4}{3}\bar{j}^c_\beta\tilde{j}^c_\beta\right)\bar{\lambda}_B\right.\\
&- && \left.\left(\frac{2}{3}\bar{\tilde{Q}}_3Q_3-\frac{1}{3}\bar{\tilde{Q}}_\alpha Q_\alpha-\frac{2}{3}\bar{\tilde{u}}^c_iu^c_i+\frac{1}{3}\bar{\tilde{d}}^c_id^c_i-\frac{5}{3}\bar{\tilde{J}}^cJ^c+\frac{4}{3}\bar{\tilde{j}}^c_\beta j^c_\beta\right)\lambda_B\right],\\
\mathcal{L}_{\tilde{q}\tilde{q}VV} &= && \frac{-1}{4}\left[g_s^2(\bar{\tilde{Q}}_i\lambda^a\lambda^b\tilde{Q}_i+\bar{\tilde{u}}^c_i\lambda^{*a}\lambda^{*b}\tilde{u}^c_i+\bar{\tilde{d}}^c_i\lambda^{*a}\lambda^{*b}\tilde{d}^c_i+\bar{\tilde{J}}^c_i\lambda^{*a}\lambda^{*b}\tilde{J}^c_i)g^a_mg^{bm}\right]\\
&- && \frac{1}{4}\left[g^2(\bar{\tilde{Q}}_3\lambda^a\lambda^b\tilde{Q}_3+\bar{\tilde{Q}}_\alpha\lambda^{*a}\lambda^{*b}\tilde{Q}_\alpha)\right]V^a_mV^{bm}-\frac{1}{2}\left[g_sg(\bar{\tilde{Q}}_3\lambda^a\lambda^b\tilde{Q}_3+\bar{\tilde{Q}}_\alpha\lambda^a\lambda^{*b}\tilde{Q}_\alpha)\right]g^a_mV^{bm}\\
&- && \frac{1}{2}g_sg'\left[\frac{2}{3}\bar{\tilde{Q}}_3\lambda^a\tilde{Q}_3-\frac{1}{3}\bar{\tilde{Q}}_\alpha\lambda^a\tilde{Q}_\alpha+\frac{2}{3}\bar{\tilde{u}}^c_i\lambda^{*a}\tilde{u}^c_i-\frac{1}{3}\bar{\tilde{d}}^c_i\lambda^{*a}\tilde{d}^c_i+\frac{5}{3}\bar{\tilde{J}}^c\lambda^{*a}\tilde{J}^c\right.\\
&- && \left.\frac{4}{3}\bar{\tilde{j}}^c_\beta\lambda^{*a}\tilde{j}^c_\beta\right]g^{am}V_m-\frac{1}{2}gg'\left[\frac{2}{3}\bar{\tilde{Q}}_3\lambda^a\tilde{Q}_3+\frac{1}{3}\bar{\tilde{Q}}_\alpha\lambda^{*a}\tilde{Q}_\alpha\right]V^{am}V_m\\
&- && \frac{1}{4}g'^2\left[\frac{4}{9}(\bar{\tilde{Q}}_3\tilde{Q}_3+\bar{\tilde{u}}^c_i\tilde{u}^c_i)+\frac{1}{9}(\bar{\tilde{Q}}_\alpha\tilde{Q}_\alpha+\bar{\tilde{d}}^c_i\tilde{d}^c_i)+\frac{25}{9}\bar{\tilde{J}}^c\tilde{J}^c+\frac{16}{9}\bar{\tilde{j}}^c_\beta\tilde{j}^c_\beta\right]V^mV_m.\\
\mathcal{L}^{quark}_{cin} &= && \tilde{Q}_i\partial^2\tilde{Q}^*_i+\tilde{u}^c_i\partial^2\tilde{u}^{c*}_i+\tilde{d}^c_i\partial^2\tilde{d}^{c*}_i+\tilde{J}^c_i\partial^2\tilde{J}^{c*}_i-iQ_i\sigma^m\partial_m\bar{Q}_i-iu^c_i\sigma^m\partial_m\bar{u}^c_i\\
&- && id^c_i\sigma^m\partial_m\bar{d}^c_i-iJ^c_i\sigma^m\partial_m\bar{J}^c_i,\\
\mathcal{L}^{quark}_F &= && |F_{Q_i}|^2+|F_{u_i}|^2+|F_{d_i}|^2+|F_{J_i}|^2,\\
\mathcal{L}^{quark}_D &= && \frac{g_s}{2}(\bar{\tilde{Q}}_i\lambda^a\tilde{Q}_i-\bar{\tilde{u}}^c_i\lambda^{*a}\tilde{u}^c_i-\bar{\tilde{d}}^c_i\lambda^{*a}\tilde{d}^c_i-\bar{\tilde{J}}^c_i\lambda^{*a}\tilde{J}^c_i)D^a_c+\frac{g}{2}\left(\bar{\tilde{Q}}_3\lambda^a\tilde{Q}_3-\bar{\tilde{Q}}_\alpha\lambda^{*a}\tilde{Q}_\alpha\right)D^a\\
&+ && \frac{g'}{2}\left[\frac{2}{3}\bar{\tilde{Q}}_3\tilde{Q}_3-\frac{1}{3}\bar{\tilde{Q}}_\alpha\tilde{Q}_\alpha-\frac{2}{3}\bar{\tilde{u}}^c_i\tilde{u}^c_i+\frac{1}{3}\bar{\tilde{d}}^c_i\tilde{d}^c_i-\frac{5}{3}\bar{\tilde{J}}^c\tilde{J}^c+\frac{4}{3}\bar{\tilde{j}}^c_\beta\tilde{j}^c_\beta\right]D.
\end{aligned}
\tag{121}
$$

A.3 Scalar Lagrangian

$$
\mathcal{L}^{Escalar}_F = |F_\eta|^2+|F_\rho|^2+|F_\chi|^2+|F_{\eta'}|^2+|F_{\rho'}|^2+|F_{\chi'}|^2,
$$

$$
\begin{aligned}
\mathcal{L}_D^{Escalar} &= \frac{g}{2}\left[\bar{\eta}\lambda^a\eta+\bar{\rho}\lambda^a\rho+\bar{\chi}\lambda^a\chi+\bar{\eta}'\lambda^{*a}\eta'-\bar{\rho}'\lambda^{*a}\rho'-\bar{\chi}'\lambda^{*a}\chi'\right]D^a \\
&+ \frac{g'}{2}\left[\bar{\rho}\rho-\bar{\chi}\chi-\bar{\rho}'\rho'+\bar{\chi}'\chi'\right]D, \\
\mathcal{L}_{Higgs} &= (\mathcal{D}_m\eta)^\dagger(\mathcal{D}^m\eta)+(\mathcal{D}_m\rho)^\dagger(\mathcal{D}^m\rho)+(\mathcal{D}_m\chi)^\dagger(\mathcal{D}^m\chi)+(\overline{\mathcal{D}_m}\eta')^\dagger(\overline{\mathcal{D}^m}\eta') \\
&+ (\overline{\mathcal{D}_m}\rho')^\dagger(\overline{\mathcal{D}^m}\rho')+(\overline{\mathcal{D}_m}\chi')^\dagger(\overline{\mathcal{D}^m}\chi')], \\
\mathcal{L}_{Higgsinos} &= i\bar{\tilde{\eta}}\bar{\sigma}^m\mathcal{D}_m\tilde{\eta}+i\bar{\tilde{\rho}}\bar{\sigma}^m\mathcal{D}_m\tilde{\rho}+i\bar{\tilde{\chi}}\bar{\sigma}^m\mathcal{D}_m\tilde{\chi}+i\bar{\tilde{\eta}}'\bar{\sigma}^m\overline{\mathcal{D}_m}\tilde{\eta}'+i\bar{\tilde{\rho}}'\bar{\sigma}^m\overline{\mathcal{D}_m}\tilde{\rho}'+i\bar{\tilde{\chi}}'\bar{\sigma}^m\overline{\mathcal{D}_m}\tilde{\chi}'], \\
\mathcal{L}_{H\tilde{H}\tilde{V}} &= -\frac{ig}{\sqrt{2}}\left[\bar{\tilde{\eta}}\lambda^a\eta\bar{\lambda}^a_A-\bar{\eta}\lambda^a\tilde{\eta}\lambda^a_A+\bar{\tilde{\rho}}\lambda^a\rho\bar{\lambda}^a_A-\bar{\rho}\lambda^a\tilde{\rho}\lambda^a_A+\bar{\tilde{\chi}}\lambda^a\chi\bar{\lambda}^a_A-\bar{\chi}\lambda^a\tilde{\chi}\lambda^a_A+\bar{\tilde{\eta}}'\lambda^{*a}\eta'\bar{\lambda}^a_A\right. \\
&+ \left.\bar{\eta}'\lambda^{*a}\tilde{\eta}'\lambda^a_A-\bar{\tilde{\rho}}'\lambda^{*a}\rho'\bar{\lambda}^a_A+\bar{\rho}'\lambda^{*a}\tilde{\rho}'\lambda^a_A-\bar{\tilde{\chi}}'\lambda^{*a}\chi'\bar{\lambda}^a_A+\bar{\chi}'\lambda^{*a}\tilde{\chi}'\lambda^a_A\right] \\
&- \frac{ig'}{\sqrt{2}}\left[\bar{\tilde{\rho}}\rho\bar{\lambda}_B-\bar{\rho}\tilde{\rho}\lambda_B-\bar{\tilde{\chi}}\chi\bar{\lambda}_B+\bar{\chi}\tilde{\chi}\lambda_B-\bar{\tilde{\rho}}'\rho'\bar{\lambda}_B+\bar{\rho}'\tilde{\rho}'\lambda_B+\bar{\tilde{\chi}}'\chi'\bar{\lambda}_B-\bar{\chi}'\tilde{\chi}'\lambda_B\right],
\end{aligned}
\tag{122}
$$

where the covariant derivative of $SU(3)$ are given by:

$$
\begin{aligned}
\mathcal{D}_m\phi_i &= \partial_m\phi_i - ig\left(\vec{V}_m.\frac{\vec{\lambda}}{2}\right)^j_i\phi_j - ig'N_{\phi_i}V_m\phi_i, \\
\overline{\mathcal{D}_m}\phi_i &= \partial_m\phi_i - ig\left(\vec{V}_m.\frac{\vec{\bar{\lambda}}}{2}\right)^j_i\phi_j - ig'N_{\phi_i}V_m\phi_i,
\end{aligned}
\tag{123}
$$

A.4 Gauge Lagrangian

Now we are dealing with $\mathcal{L}_{Gauge}$ that can be expanded as

$$
\mathcal{L}_{Gauge} = \mathcal{L}_{dc} + \mathcal{L}_D^{gauge}. \tag{124}
$$

where

$$
\begin{aligned}
\mathcal{L}_D^{gauge} &= \frac{1}{2}D^a_C D^a_C + \frac{1}{2}D^a D^a + \frac{1}{2}DD, \\
\mathcal{L}_{dc} &= -\frac{1}{4}\left(G^{amn}G^a_{mn} + W^{amn}W^a_{mn} + F^{mn}F_{mn}\right) \\
&- \imath\left(\bar{\lambda}^a_C\bar{\sigma}^n\mathcal{D}^C_n\lambda^a_C + \bar{\lambda}^a_A\bar{\sigma}^n\mathcal{D}^L_n\lambda^a_A + \bar{\lambda}_B\bar{\sigma}^n\partial_n\lambda_B\right) \quad ,
\end{aligned}
\tag{125}
$$

with

$$
G^a_{mn} = \partial_m g^a_n - \partial_n g^a_m - g f^{abc} g^b_m g^c_n,
$$

$$
\begin{aligned}
W^a_{mn} &= \partial_m V^a_n - \partial_n V^a_m - g f^{abc} V^b_m V^c_n, \\
B_{mn} &= \partial_m V_n - \partial_n V_m, \\
\mathcal{D}^C_n \lambda^a_C &= \partial_n \lambda^a_C - g_s f^{abc} g^b_m \lambda^c_C, \\
\mathcal{D}^L_n \lambda^a_A &= \partial_n \lambda^a_A - g f^{abc} V^b_m \lambda^c_A, \qquad (126)
\end{aligned}
$$

f^{abc} are the constant structure of $SU(3)$ gauge group.

A.5 Superpotential Lagrangian

$$
\begin{aligned}
W_2 &= \mathcal{L}^{W2}_F + \mathcal{L}_{\tilde{\eta}L} + \mathcal{L}_{HMT}, \\
W_3 &= \mathcal{L}^{W3}_F + \mathcal{L}_{ll\tilde{l}} + \mathcal{L}_{llH} + \mathcal{L}_{l\tilde{l}\tilde{H}} + \mathcal{L}_{l\tilde{H}H} + \mathcal{L}_{\tilde{l}HH} + \mathcal{L}_{H\tilde{H}\tilde{H}} + \mathcal{L}_{qqH} + \mathcal{L}_{q\tilde{q}\tilde{H}} \\
&+ \mathcal{L}_{lq\tilde{q}} + \mathcal{L}_{\tilde{l}q\tilde{q}} + \mathcal{L}_{qq\tilde{q}}, \qquad (127)
\end{aligned}
$$

As componentes de cada lagrangiana são escritas como

$$
\begin{aligned}
\mathcal{L}^{W2}_F &= \frac{\mu_0}{2}\left(\tilde{L}F_{\eta'} + \eta' F_L\right) + \frac{\mu_\eta}{2}\left(\eta F_{\eta'} + \eta' F_\eta\right) + \frac{\mu_\rho}{2}\left(\rho F_{\rho'} + \rho' F_\rho\right) + \frac{\mu_\chi}{2}\left(\chi F_{\chi'} + \chi' F_\chi\right), \\
\mathcal{L}_{\tilde{\eta}L} &= -\frac{\mu_0}{2} L\tilde{\eta}'; \;\; \mathcal{L}_{HMT} = -\frac{\mu_\eta}{2}\tilde{\eta}_i\tilde{\eta}'_i - \frac{\mu_\rho}{2}\tilde{\rho}_i\tilde{\rho}'_i - \frac{\mu_\chi}{2}\tilde{\chi}_i\tilde{\chi}'_i, \\
\mathcal{L}_F &= \frac{1}{3}[3\lambda_1\epsilon F_L\tilde{L}\tilde{L} + \lambda_2\epsilon(2F_L\eta + F_\eta\tilde{L})\tilde{L} + f_1\epsilon(F_\rho\chi\eta + \rho F_\chi\eta + \rho\chi F_\eta) \\
&+ f'_1\epsilon(F_{\rho'}\chi'\eta' + \rho' F_{\chi'}\eta' + \rho'\chi' F_{\eta'}) + \kappa_1(F_{Q_1}\eta'\tilde{u}^c_i + \tilde{Q}_1 F_{\eta'}\tilde{u}^c_i + \tilde{Q}_1\eta' F_{u_i}) \\
&+ \kappa_2(F_{Q_1}\rho'\tilde{d}^c_i + \tilde{Q}_1 F_{\rho'}\tilde{d}^c_i + \tilde{Q}_1\rho' F_{d_i}) + \kappa_3(F_{Q_1}\chi'\tilde{J}^c + \tilde{Q}_1 F_{\chi'}\tilde{J}^c + \tilde{Q}_1\chi' F_J) \\
&+ \kappa_4(F_{Q_\alpha}\eta\tilde{d}^c_i + \tilde{Q}_\alpha F_\eta\tilde{d}^c_i + \tilde{Q}_\alpha\eta F_{d_i}) + \kappa_5(F_{Q_\alpha}\rho\tilde{u}^c_i + \tilde{Q}_\alpha F_\rho\tilde{u}^c_i + \tilde{Q}_\alpha\rho F_{u_i}) \\
&+ \kappa_6(F_{Q_\alpha}\chi\tilde{j}^c_\beta + \tilde{Q}_\alpha F_\chi\tilde{j}^c_\beta + \tilde{Q}_\alpha\chi F_{j_\beta}) + \kappa_7(F_{Q_\alpha}\tilde{L}\tilde{d}^c_i + \tilde{Q}_\alpha F_L\tilde{d}^c_i + \tilde{Q}_\alpha\tilde{L}F_{d_i}) \\
&+ \xi_1(2F_{d_i}\tilde{d}^c_j\tilde{u}^c_k + \tilde{d}^c_i\tilde{d}^c_j F_{u_k}) + \xi_2(2F_{u_i}\tilde{u}^c_j\tilde{j}^c_\beta + \tilde{u}^c_i\tilde{u}^c_j F_{j_\beta}) \\
&+ \xi_3(F_{d_i}\tilde{J}^c\tilde{j}^c_\beta + \tilde{d}^c_i F_J\tilde{j}^c_\beta + \tilde{d}^c_i\tilde{J}^c F_{j_\beta}) + \lambda_4\epsilon(F_L\chi\rho + \tilde{L}F_\chi\rho + \tilde{L}\chi F_\rho)], \\
\mathcal{L}_{ll\tilde{l}} &= -\frac{\lambda_1}{3}\epsilon(LL\tilde{L} + \tilde{L}LL + L\tilde{L}L); \;\; \mathcal{L}_{llH} = -\frac{1}{3}\lambda_2\epsilon LL\eta, \\
\mathcal{L}_{l\tilde{l}\tilde{H}} &= -\frac{1}{3}\lambda_2\epsilon(\tilde{L}L\tilde{\eta} + L\tilde{L}\tilde{\eta}); \;\; \mathcal{L}_{l\tilde{H}H} = -\frac{\lambda_4}{3}\epsilon(L\tilde{\chi}\rho + L\chi\tilde{\rho}), \\
\mathcal{L}_{H\tilde{H}\tilde{H}} &= -\frac{1}{3}[f_1\epsilon(\tilde{\rho}\tilde{\chi}\eta + \rho\tilde{\chi}\tilde{\eta} + \tilde{\rho}\chi\tilde{\eta}) + f'_1\epsilon(\tilde{\rho}'\tilde{\chi}'\eta' + \rho'\tilde{\chi}'\tilde{\eta}' + \tilde{\rho}'\chi'\tilde{\eta}')], \\
\mathcal{L}_{qqH} &= -\frac{1}{3}[\kappa_1 Q_1\eta' u^c_i + \kappa_2 Q_1\rho' d^c_i + \kappa_3 Q_1\chi' J^c + \kappa_4 Q_\alpha\eta d^c_i + \kappa_5 Q_\alpha\rho u^c_i + \kappa_6 Q_\alpha\chi j^c_\beta], \\
\mathcal{L}_{q\tilde{q}\tilde{H}} &= -\frac{1}{3}[\kappa_1(Q_1\tilde{u}^c_i + \tilde{Q}_1 u^c_i)\tilde{\eta}' + \kappa_2(Q_1\tilde{d}^c_i + \tilde{Q}_1 d^c_i)\tilde{\rho}' + \kappa_3(Q_1\tilde{J}^c + \tilde{Q}_1 J^c)\tilde{\chi}'
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \kappa_4(Q_\alpha\tilde{d}^c_i + \tilde{Q}_\alpha d^c_i)\tilde{\eta} + \kappa_5(Q_\alpha\tilde{u}^c_i + \tilde{Q}_\alpha u^c_i)\tilde{\rho} + \kappa_6(Q_\alpha\tilde{j}^c_\beta + \tilde{Q}_\alpha j^c_\beta)\tilde{\chi}], \\
\mathcal{L}_{lq\tilde{q}} &= -\frac{\kappa_7}{3}(Q_\alpha\tilde{d}^c_i + \tilde{Q}_\alpha d^c_i)L;\ \ \mathcal{L}_{\tilde{l}qq} = -\frac{\kappa_7}{3}Q_\alpha\tilde{L}d^c_i,\ \ \mathcal{L}_{\tilde{l}HH} = -\frac{\lambda_4}{3}\tilde{L}\chi\rho, \\
\mathcal{L}_{qq\tilde{q}} &= -\frac{1}{3}[\xi_1(d^c_i d^c_j\tilde{u}^c_k + \tilde{d}^c_i d^c_j u^c_k + d^c_i\tilde{d}^c_j u^c_k) + \xi_2(u^c_i u^c_j\tilde{j}^c_\beta + \tilde{u}^c_i u^c_j j^c_\beta + u^c_i\tilde{u}^c_j j^c_\beta) \\
&+ \quad \xi_3(d^c_i J^c\tilde{j}^c_\beta + \tilde{d}^c_i J^c j^c_\beta + d^c_i\tilde{J}^c j^c_\beta)].
\end{aligned}
\tag{128}
$$

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Vertex	coupling constant/e
γW^+W^-	1
ZW^+W^-	$1/t_W$
γV^+V^-	1
ZV^+V^-	$-(1+2s_W^2)/\sin 2\theta_W$
$\gamma U^{++}U^{--}$	2
$ZU^{++}U^{--}$	$(1-4s_W^2)/\sin 2\theta_W$
$Z'V^+V^-$	$-\sqrt{3(1-4s_W^2)}/\sin 2\theta_W$
$Z'U^{++}U^{--}$	$-\sqrt{3(1-4s_W^2)}/\sin 2\theta_W$
$U^{--}V^+W^+$	$1/(\sqrt{2}\ s_W)$
$U^{++}W^-V^-$	$1/(\sqrt{2}\ s_W)$
$W_\mu^+W_\nu^-W_\alpha^+W_\beta^-$	$S_{\mu\alpha,\nu\beta}$
$V_\mu^+V_\nu^-V_\alpha^+V_\beta^-$	$S_{\mu\alpha,\nu\beta}$
$U_\mu^{++}U_\nu^{--}U_\alpha^{++}U_\beta^{--}$	$S_{\mu\alpha,\nu\beta}$
$W_\mu^+W_\nu^-V_\alpha^+V_\beta^-$	$S_{\mu\beta,\nu\alpha}/2$
$W_\mu^+W_\nu^-U_\alpha^{++}U_\beta^{--}$	$S_{\mu\alpha,\nu\beta}/2$
$V_\mu^+V_\nu^-U_\alpha^{++}U_\beta^{--}$	$S_{\mu\alpha,\nu\beta}/2$
$\gamma_\mu\gamma_\nu W_\alpha^+W_\beta^-$	$-s_W^2 S_{\mu\nu,\alpha\beta}$
$\gamma_\mu\gamma_\nu V_\alpha^+V_\beta^-$	$-s_W^2 S_{\mu\nu,\alpha\beta}$
$\gamma_\mu\gamma_\nu U_\alpha^{++}U_\beta^{--}$	$-4s_W^2 S_{\mu\nu,\alpha\beta}$
$Z_\mu Z_\nu W_\alpha^+W_\beta^-$	$-c_W^2 S_{\mu\nu,\alpha\beta}$
$Z_\mu Z_\nu V_\alpha^+V_\beta^-$	$-(c_W-3s_Wt_W)^2 S_{\mu\nu,\alpha\beta}/4$
$Z_\mu Z_\nu U_\alpha^{++}U_\beta^{--}$	$-(c_W-3s_Wt_W)^2 S_{\mu\nu,\alpha\beta}/4$
$Z'_\mu Z'_\nu V_\alpha^+V_\beta^-$	$-3(1-3t_W^2)S_{\mu\nu,\alpha\beta}/4$
$Z'_\mu Z'_\nu U_\alpha^{++}U_\beta^{--}$	$-3(1-3t_W^2)S_{\mu\nu,\alpha\beta}/4$
$\gamma_\mu Z_\nu W_\alpha^+W_\beta^-$	$-c_Ws_W S_{\mu\nu,\alpha\beta}$
$\gamma_\mu Z_\nu V_\alpha^+V_\beta^-$	$s_W(c_W+3s_Wt_W)S_{\mu\nu,\alpha\beta}/2$
$\gamma_\mu Z_\nu U_\alpha^{++}U_\beta^{--}$	$-s_W(c_W-3s_Wt_W)S_{\mu\nu,\alpha\beta}$
$\gamma_\mu Z'_\nu V_\alpha^+V_\beta^-$	$s_W\sqrt{(3-9t_W^2)}S_{\mu\nu,\alpha\beta}/2$
$\gamma_\mu Z'_\nu U_\alpha^{++}U_\beta^{--}$	$s_W\sqrt{(3-9t_W^2)}S_{\mu\nu,\alpha\beta}$
$Z_\mu Z'_\nu V_\alpha^+V_\beta^-$	$-(c_W+3s_Wt_W)\sqrt{(3-9t_W^2)}S_{\mu\nu,\alpha\beta}/4$
$Z_\mu Z'_\nu U_\alpha^{++}U_\beta^{--}$	$(c_W-3s_Wt_W)\sqrt{(3-9t_W^2)}S_{\mu\nu,\alpha\beta}/4$
$Z'_\mu W_\nu^+V_\alpha^+U_\beta^{--}$	$\sqrt{6(1-3t_W^2)}S_{\mu\nu,\alpha\beta}/4$
$\gamma_\mu W_\nu^+V_\alpha^+U_\beta^{--}$	$3s_W V_{\mu\nu\alpha\beta}/\sqrt{2}$
$Z_\mu W_\nu^+V_\alpha^+U_\beta^{--}$	$3\left(s_Wt_W S_{\mu\nu,\alpha\beta}+c_W U_{\mu\beta\nu\alpha}\right)/(2\sqrt{2})$

Table 3: Trilinear and Quartic couplings in the MSUSY331